

1. Řešte soustavy rovnic:

$$(a) \quad \begin{aligned} x + 2y &= 5 \\ y - 3z &= 5 \\ 3x - z &= 4 \end{aligned}$$

$$(e) \quad \begin{aligned} x + 2y &= 5 \\ y - 3z &= 5 \\ 3x - z &= 4 \end{aligned}$$

$$(b) \quad \begin{aligned} 4x + 3y &= 4 \\ 2x + 2y - 2z &= 0 \\ 5x + 3y + z &= -2 \end{aligned}$$

$$(f) \quad \begin{aligned} 2x - y + z &= 0 \\ x + 2y - 2z &= 0 \\ 3x + y - z &= 0 \end{aligned}$$

$$(c) \quad \begin{aligned} x - 2y - 3z &= 0 \\ 3x + 2y - z &= 0 \\ 3y + z &= 0 \end{aligned}$$

$$(g) \quad \begin{aligned} x + y + z &= 4 \\ x + 2y + 4z &= 12 \\ -x + 2z &= 4 \end{aligned}$$

$$(d) \quad \begin{aligned} x + 2y + 3z &= 14 \\ 3x + 2y + z &= 10 \\ x - 2y - 5z &= 15 \end{aligned}$$

$$(h) \quad \begin{aligned} -x + 3y &= -72 \\ 3x + 4y - 4z &= -4 \\ -20x - 12y + 5z &= -50 \end{aligned}$$

2. Řešte soustavy rovnic:

$$(a) \quad \begin{aligned} 2x_1 - 3x_2 + 5x_3 + 7x_4 &= 1 \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 &= 2 \\ 2x_1 - 3x_2 - 11x_3 - 15x_4 &= 1 \end{aligned}$$

$$(e) \quad \begin{aligned} x_1 + x_2 + x_3 + x_4 - 4x_5 &= 10 \\ 2x_1 - x_2 + x_3 + 3x_4 - 5x_5 &= 15 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 2x_5 &= -10 \\ 3x_1 + x_2 - 2x_3 - x_4 - x_5 &= -5 \end{aligned}$$

$$(b) \quad \begin{aligned} 2x_1 + 5x_2 - 8x_3 &= 8 \\ 4x_1 + 3x_2 - 9x_3 &= 9 \\ 2x_1 + 3x_2 - 5x_3 &= 7 \\ x_1 + 8x_2 - 7x_3 &= 12 \end{aligned}$$

$$(f) \quad \begin{aligned} 2x_1 + 3x_2 - x_3 + 2x_4 &= 3 \\ 5x_1 + 7x_2 - 4x_3 + 7x_4 &= 8 \\ x_1 + 2x_2 + x_3 - x_4 &= 1 \\ 4x_1 + 7x_2 + x_3 &= 5 \end{aligned}$$

$$(c) \quad \begin{aligned} 2x_1 - x_2 + 3x_3 - 2x_4 &= 4 \\ 8x_1 + 5x_2 + 9x_3 &= -2 \\ 5x_1 + 2x_2 + 6x_3 - x_4 &= -3 \end{aligned}$$

$$(g) \quad \begin{aligned} x_1 + 3x_2 - x_3 - x_4 &= 4 \\ 2x_1 + 4x_2 - x_3 + x_4 &= -1 \\ 3x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 - x_2 + x_3 - 2x_4 &= 7 \end{aligned}$$

$$(d) \quad \begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\ 2x_1 + 3x_2 + 4x_3 + x_4 &= 2 \\ 3x_1 + 4x_2 + x_3 + 2x_4 &= -2 \\ 4x_1 + x_2 + 2x_3 + 3x_4 &= 2 \end{aligned}$$

$$(h) \quad \begin{aligned} x_1 + 2x_2 + 3x_3 - x_4 &= 0 \\ x_1 - x_2 + x_3 + 2x_4 &= 4 \\ x_1 + 5x_2 + 5x_3 - 4x_4 &= -4 \\ x_1 + 8x_2 + 7x_3 - 7x_4 &= -8 \end{aligned}$$

<p>(i)</p> $\begin{aligned}x_1 + 3x_2 + x_3 - x_4 &= 2 \\2x_1 - 2x_2 + x_4 &= -3 \\2x_1 + 3x_2 + x_3 - 3x_4 &= -6 \\3x_1 + 4x_2 - x_3 + 2x_4 &= 0\end{aligned}$	<p>(k)</p> $\begin{aligned}2x_1 + x_2 + x_3 + x_4 &= 0 \\x_1 + 2x_2 + x_3 + x_4 &= 0 \\x_1 + x_2 + 2x_3 + x_4 &= 0 \\x_1 + x_2 + x_3 + 2x_4 &= 0 \\x_1 + x_2 + x_3 + x_4 &= 0\end{aligned}$
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<p>(j)</p> $\begin{aligned}2x_1 - 4x_2 + 5x_3 + 3x_4 &= 0 \\3x_1 - 6x_2 + 4x_3 + 2x_4 &= 0 \\4x_1 - 8x_2 + 17x_3 + 11x_4 &= 0\end{aligned}$	<p>(l)</p> $\begin{aligned}2x_1 + 5x_2 - 4x_3 - 4x_4 + 4x_5 &= 0 \\2x_1 + x_2 - x_3 - x_4 + x_5 &= 0 \\3x_1 + 8x_2 - 6x_3 - 6x_4 + 6x_5 &= 0 \\3x_1 - x_2 - 2x_3 + x_4 - x_5 &= 0\end{aligned}$
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3. Pokud je to možné, řešte následující soustavy Cramerovým pravidlem, jinak je vyřešte pomocí GEM:

<p>(a)</p> $\begin{aligned}2x_1 + 3x_2 &= 1 \\x_2 - x_3 &= -1 \\-x_1 + 2x_2 &= 2\end{aligned}$	<p>(d)</p> $\begin{aligned}3x_1 + 6x_2 - 9x_3 &= 3 \\7x_1 + 14x_2 - 21x_3 &= 7 \\5x_1 + 10x_2 + 15x_3 &= 5 \\x_1 + 2x_2 - 3x_3 &= 1\end{aligned}$
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<p>(b)</p> $\begin{aligned}x_1 + x_2 + 3x_3 &= 7 \\x_1 - 3x_2 + 2x_3 &= 5 \\x_1 + x_2 + x_3 &= 3\end{aligned}$	<p>(e)</p> $\begin{aligned}2x_1 + x_2 + x_3 &= 0 \\x_1 + 2x_2 + x_3 &= 0 \\x_1 + x_2 + 2x_3 &= 0\end{aligned}$
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<p>(c)</p> $\begin{aligned}2x_1 - x_2 + 2x_3 &= -4 \\4x_1 + x_2 + 4x_3 &= -2 \\x_1 + x_2 + 2x_3 &= -1\end{aligned}$	<p>(f)</p> $\begin{aligned}2x_1 + x_2 + x_3 + x_4 &= 0 \\x_1 + 2x_2 + x_3 + x_4 &= 0 \\x_1 + x_2 + 2x_3 + x_4 &= 0\end{aligned}$
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4. V závislosti na parametru a řešte soustavy rovnic:

<p>(a)</p> $\begin{aligned}2x + y + az &= 4 \\x + z &= 2 \\x + y + z &= 2\end{aligned}$	<p>(c)</p> $\begin{aligned}x + y + 2t &= 3 \\3x - y + z - t &= 1 \\5x - 3y + 2z - 4t &= a \\2x + y + z + t &= 2\end{aligned}$
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<p>(b)</p> $\begin{aligned}x + y + z &= a \\x + (a + 1)y + z &= 2a \\x + y + (1 + a)z &= 0\end{aligned}$	<p>(d)</p> $\begin{aligned}x + y + z &= 0 \\x - y + z &= 0 \\ax + z &= 0\end{aligned}$
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Řešení:

1. (a) $(1, 2, -1)$ (e) $(1, 2, -1)$
 (b) $(-11, 16, 5)$ (f) $(0, t, t)$
 (c) $(0, 0, 0)$ (g) $(2t - 4, 8 - 3t, t)$ pro $t \in \mathbb{R}$
 (d) nemá řešení (h) $(12, -20, -10)$
2. (a) $(\frac{3}{2}s - \frac{1}{16}t + \frac{1}{2}, s, -\frac{11}{8}t, t)$ pro $s, t \in \mathbb{R}$ (g) $(1 - t, t, 2t - 3)$ pro $t \in \mathbb{R}$
 (b) $(3, 2, 1)$ (h) $(5s - 7t + 6, 2s, -3s + 3t - 2, 2t)$ pro $s, t \in \mathbb{R}$
 (c) nemá řešení (i) $(-2, 1, 4, 3)$
 (d) $(1, -1, 1, -1)$ (j) $(2s - 2t, s, 5t, -7t)$ pro $s, t \in \mathbb{R}$
 (e) $(1 + t, 2 + t, 3 + t, 4 + t, t)$ pro $t \in \mathbb{R}$ (k) $(0, 0, 0, 0)$
 (f) $(3 + 5s - 7t, -1 - 3s + 4t, s, t)$ pro $s, t \in \mathbb{R}$ (l) $(0, 0, 0, t, t)$ pro $t \in \mathbb{R}$
3. (a) Cramerovo pravidlo: $(-\frac{4}{7}, \frac{5}{7}, \frac{12}{7})$ (d) GEM: $(1 - 2t, t, 0)$ pro $t \in \mathbb{R}$
 (b) Cramerovo pravidlo: $(1, 0, 2)$ (e) Cramerovo pravidlo: $(0, 0, 0)$
 (c) Cramerovo pravidlo: $(1, 2, -2)$ (f) GEM: $(2t, t, -t, 4t)$ pro $t \in \mathbb{R}$
4. (a) $(2, 0, 0)$ pro $a \neq 2$,
 $(2 - t, 0, t)$ pro $a = 2, t \in \mathbb{R}$ (c) nemá řešení pro $a \neq -5$,
 $(1 - \frac{4t}{3}, 2 - \frac{4t}{3}, \frac{5t}{3} - 2, t)$ pro $a = -5$,
 $t \in \mathbb{R}$
 (b) $(a, 1, -1)$ pro $a \neq 0$,
 $(1 - s - t, s, t)$ pro $a = 0, t, s \in \mathbb{R}$ (d) $(0, 0, 0)$ pro $a \neq 1$,
 $(-t, 0, t)$ pro $a = -1$