8. minitest - varianta A

Limity funkcí

12. 12. 2023

Vypočtěte následující limity.

$a)\ \lim\limits_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9} \qquad b)\ \lim\limits_{x\to\infty} \left(6\log_2\left(\frac{8x+3}{16x+1}\right) + 2\sqrt{\frac{9x^2+3x+1}{x^2+3x+9}}\right)$

a) $\lim\limits_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim\limits_{x\to 3} \frac{(x+1)-4}{(x^2-9)(\sqrt{x+1}+2)}$

$= \lim\limits_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)(\sqrt{x+1}+2)} = \frac{1}{6\cdot(\sqrt{4}+2)} = \frac{1}{24}$

b) $\lim\limits_{x\to\infty} \left(6\log_2\left(\frac{8x+3}{16x+1}\right) + 2\cdot\sqrt{\frac{9x^2+3x+1}{x^2+3x+9}}\right) =$

$= 6\log_2\left(\underbrace{\lim\limits_{x\to\infty} \frac{\cancel{x}(8+\frac{3}{x})}{\cancel{x}(16+\frac{1}{x})}}_{\frac{8+0}{16+0}=\frac{1}{2}}\right) + 2\cdot\sqrt{\underbrace{\lim\limits_{x\to\infty} \frac{\cancel{x^2}(9+\frac{3}{x}+\frac{1}{x^2})}{\cancel{x^2}(1+\frac{3}{x}+\frac{9}{x^2})}}_{\frac{9+0+0}{1+0+0}=9}}$

$= \underbrace{6\log_2\frac{1}{2}}_{-1} + 2\cdot\sqrt{9} = 6\cdot(-1)+2\cdot 3 = 0$

8. minitest - varianta B

Limity funkcí

12. 12. 2023

Vypočtěte následující limity.

$$a)\ \lim_{x\to 4} \frac{\sqrt{x}-2}{x^2+x-20} \qquad b)\ \lim_{x\to\infty} \left(\frac{2^{x+2}+1}{2^{x-1}+3} + 8\log_4\left(\frac{4x+9}{16x+8}\right)\right)$$

$$a)\ \lim_{x\to 4} \frac{\sqrt{x}-2}{x^2+x-20} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(x+5)(\sqrt{x}+2)}$$

$$= \frac{1}{(4+5)(\sqrt{4}+2)} = \frac{1}{9\cdot 4} = \frac{1}{36}$$

$$b)\ \lim_{x\to\infty}\left(\frac{2^{x+2}+1}{2^{x-1}+3} + 8\cdot\log_4\left(\frac{4x+9}{16x+8}\right)\right) =$$

$$= \underbrace{\lim_{x\to+\infty} \frac{\cancel{2^x}\cdot\left(4+\frac{1}{2^x}\right)}{\cancel{2^x}\cdot\left(\frac{1}{2}+\frac{3}{2^x}\right)}}_{=\frac{4+0}{\frac{1}{2}+0}=8} + 8\cdot\log_4\Bigg(\underbrace{\lim_{x\to\infty}\frac{\cancel{x}\left(4+\frac{9}{x}\right)}{\cancel{x}\cdot\left(16+\frac{8}{x}\right)}}_{\frac{4+0}{16+0}=\frac{1}{4}}\Bigg)$$

$$= 8 + 8\cdot\underbrace{\log_4\frac{1}{4}}_{-1} = 8+8\cdot(-1) = \underline{\underline{0}}$$