8. minitest - varianta A

Konvergence řady

8. 12. 2023

Rozhodněte o konvergenci řady

$$\sum_{n=1}^{\infty}\left(\frac{3^n-3^{n-1}+3^{n-2}}{2^n+3^n+3^{n-1}-3^{n-2}}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\frac{3^n-3^{n-1}+3^{n-2}}{2^n+3^n+3^{n-1}-3^{n-2}}=\lim_{n\to\infty}\frac{\cancel{3^n}\cdot(1-3^{-1}+3^{-2})}{\cancel{3^n}\cdot\left(\left(\frac{2}{3}\right)^n+1+3^{-1}-3^{-2}\right)}$$

$$=\frac{1-\frac{1}{3}+\frac{1}{9}}{0+1+\frac{1}{3}-\frac{1}{9}}=\frac{\frac{9-3+1}{9}}{\frac{9+3-1}{9}}=\frac{\frac{7}{9}}{\frac{11}{9}}=\underline{\underline{\frac{7}{11}}}\in[0,1)$$

$$\left(\frac{2}{3}\right)^n\xrightarrow[n\to\infty]{}0$$

$\Rightarrow$ Řada $\sum_{n=1}^{\infty}a_n$ konverguje

8. minitest - varianta B

Konvergence řady

8. 12. 2023

Rozhodněte o konvergenci řady

$$\sum_{n=1}^{\infty}\left(\frac{2^{2n+1}+3^n+\sqrt{2^{4n+1}}}{2^{n+1}+3^{n+1}+4^{n+1}}\right)^n$$

$$\left[\begin{array}{l} 2^{2n+1} = 2\cdot 4^n \\ (2^{4n+1})^{\frac{1}{2}} = \sqrt{2}\cdot 4^n \end{array}\right]$$

$$\lim_{n\to+\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{2^{2n+1}+3^n+(2^{4n+1})^{\frac{1}{2}}}{2^{n+1}+3^{n+1}+4^{n+1}} =$$

$$= \lim_{n\to\infty}\frac{\cancel{4^n}\cdot\left(2+\left(\frac{3}{4}\right)^n+\sqrt{2}\right)}{\underbrace{4^{n+1}}_{\cancel{4^n}\cdot 4^1}\cdot\left(\left(\frac{2}{4}\right)^{n+1}+\left(\frac{3}{4}\right)^{n+1}+1\right)} = \frac{2+0+\sqrt{2}}{4\cdot(0+0+1)} = \frac{2+\sqrt{2}}{4} \in [0,1)$$

$$\boxed{q^n \xrightarrow[n\to\infty]{} 0 \text{ pro } |q|<1}$$

$$\Rightarrow \sum_{n=1}^{\infty} a_n \text{ konverguje}$$