Varianta A

1) $\int_0^1 \frac{(x-1)^2}{\sqrt{x}}\,dx = \int_0^1 \frac{x^2-2x+1}{\sqrt{x}}\,dx = \int_0^1 \left(x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx =$

$= \left[\frac{2}{5}x^{\frac{5}{2}} - 2\cdot\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right]_0^1 = \frac{2}{5} - \frac{4}{3} + 2 = \frac{6-20+30}{15} = \underline{\underline{\frac{16}{15}}}$

$\int \ln x\,dx \overset{P.P.}{=} x\ln x - \int \frac{1}{x}\cdot x\,dx = \underline{\underline{x\ln x - x + c}}$

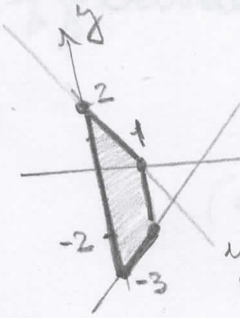
$\left| \begin{array}{ll} u = \ln x & v' = 1 \\ u' = \frac{1}{x} & v = x \end{array} \right|$

$\int \frac{x}{\sqrt[3]{x^2+1}}\,dx = \frac{1}{2}\int \frac{dt}{\sqrt[3]{t}} = \frac{1}{2}\int t^{-\frac{1}{3}}\,dt = \frac{1}{2}\cdot\frac{3}{2}\cdot t^{\frac{2}{3}} =$

$\left| \begin{array}{l} x^2+1 = t \\ 2x\,dx = dt \end{array} \right|$

$= \underline{\underline{\frac{3}{4}\cdot\sqrt[3]{(x^2+1)^2} + c}}$

2)



$y = x-3$

$y = -2x+2$

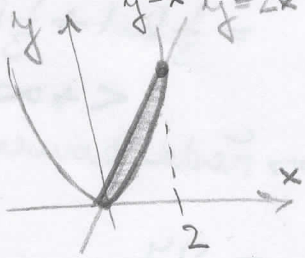
$\int_0^1 \int_{x-3}^{-2x+2} 1\,dy\,dx = \int_0^1 \left(-2x+2-(x-3)\right)dx$

$= \int_0^1 (-3x+5)\,dx = \left[-3\frac{x^2}{2} + 5x\right]_0^1$

$= -\frac{3}{2} + 5 = \underline{\underline{\frac{7}{2}}}$

3) $\iint_M x\,dy\,dx = \int_0^2 \int_{x^2}^{2x} x\,dy\,dx = \int_0^2 x\cdot\left[y\right]_{x^2}^{2x}\,dx =$

$= \int_0^2 x\cdot(2x - x^2)\,dx = \int_0^2 (2x^2 - x^3)\,dx$



$2x = x^2$

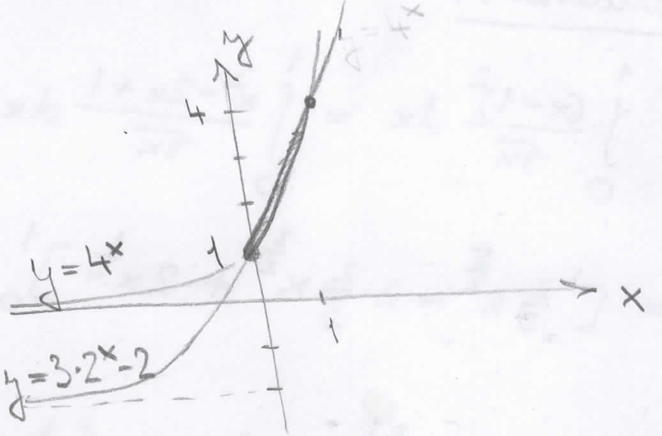
$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \vee x = 2$

$= \left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \frac{16}{3} - 4 = \underline{\underline{\frac{4}{3}}}$

4) $M: 4^x \le y \le 3\cdot 2^x - 2$



$4^x = 3\cdot 2^x - 2$ $\quad \boxed{2^x = t}$

$t^2 = 3t - 2$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 2 \lor t = 1$

$x = 1 \lor x = 0$

$$\iint_M f(x,y)\,dy\,dx = \int_0^1 \int_{4^x}^{3\cdot 2^x - 2} f(x,y)\,dy\,dx = \int_1^4 \int_{\log_2\left(\frac{y+2}{3}\right)}^{\log_4 y} f(x,y)\,dx\,dy$$

5) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n}$ konverguje $\iff \int_1^{\infty} \frac{1}{x^2+3x}\,dx$ konverguje, neboť:

$f(x) := \frac{1}{x^2+3x}$ splňuje:
- $f \ge 0$ na $\langle 1, +\infty)$
- f je spojitá na $\langle 1, \infty)$
- $\lim_{x\to\infty} f(x) = 0$
- f je klesající na $\langle 1, \infty)$ $(f' < 0)$

$$\int_1^{\infty} \frac{1}{x^2+3x}\,dx = \frac{1}{3}\cdot\int_1^{\infty}\left(\frac{1}{x} - \frac{1}{x+3}\right)dx = \frac{1}{3}\cdot\Big[\ln|x| - \ln|x+3|\Big]_1^{\infty}$$
$$= \frac{1}{3}\cdot\Big[\ln\left(\frac{x}{x+3}\right)\Big]_1^{\infty} = \frac{1}{3}\cdot\lim_{x\to\infty}\ln\left(\frac{x}{x+3}\right) - \frac{1}{3}\cdot\ln\frac{1}{4}$$
$$= \frac{1}{3}\ln 1 + \frac{1}{3}\ln 4 < +\infty$$

$\Rightarrow$ řada konverguje

$$\frac{1}{x^2+3x} = \frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} \quad \Big|\cdot x(x+3)$$

$1 = A(x+3) + Bx$

$x=0: 1 = 3A \Rightarrow A = \frac{1}{3}$

$x=-3: 1 = -3B \Rightarrow B = -\frac{1}{3}$

6) $9 + 6 + 4 + \frac{8}{3} + \frac{16}{9} + \dots = \frac{9}{1-\frac{2}{3}} = \frac{9}{\frac{1}{3}} = \underline{\underline{27}}$