Varianta B

1) $$\int_0^1 x(\sqrt{x}-1)^2\,dx = \int_0^1 x(x-2\sqrt{x}+1)\,dx = \int_0^1 (x^2-2x^{\frac{3}{2}}+x)\,dx$$

$$= \left[\frac{x^3}{3} - 2\cdot\frac{2}{5}\cdot x^{\frac{5}{2}} + \frac{x^2}{2}\right]_0^1 = \frac{1}{3}-\frac{4}{5}+\frac{1}{2} = \frac{10-24+15}{30} = \underline{\underline{\frac{1}{30}}}$$

$$\int x\ln x\,dx \overset{P.P.}{=} \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \underline{\underline{\frac{x^2}{2}\ln x - \frac{x^2}{4} + C}}$$

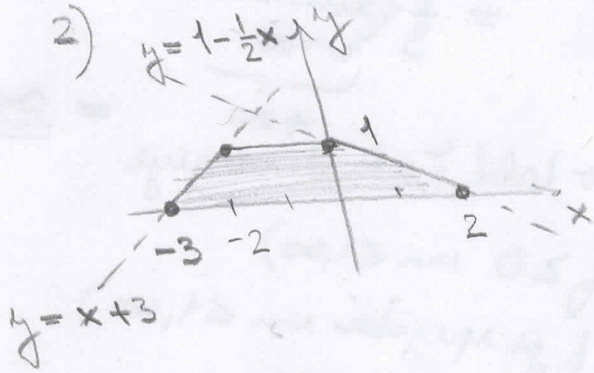
$$\left| \begin{matrix} u = \ln x & v' = x \\ u' = \frac{1}{x} & v = \frac{x^2}{2} \end{matrix} \right|$$

$$\int x\sqrt{x+3}\,dx = \int (t-3)\sqrt{t}\,dt = \int (t^{\frac{3}{2}} - 3t^{\frac{1}{2}})\,dt =$$

$$\left| \begin{matrix} x+3 = t \\ dx = dt \end{matrix} \right| \Rightarrow x = t-3$$

$$= \frac{2}{5}t^{\frac{5}{2}} - 3\cdot\frac{2}{3}t^{\frac{3}{2}} + C = \underline{\underline{\frac{2}{5}(x+3)^{\frac{5}{2}} - 2\cdot(x+3)^{\frac{3}{2}} + C}}$$

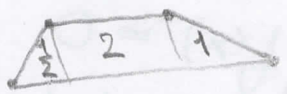
2)



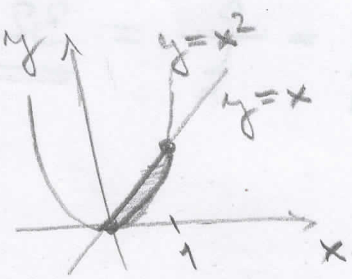
$$\int_0^1\int_{y-3}^{2-2y} 1\,dx\,dy = \int_0^1 (2-2y-y+3)\,dy$$

$$= \int_0^1 (5-3y)\,dy = \left[5y - 3\frac{y^2}{2}\right]_0^1$$

$$= 5 - \frac{3}{2} = \underline{\underline{\frac{7}{2}}}$$



3) $$\iint_M \sqrt{xy}\,dy\,dx = \int_0^1\int_{x^2}^{x} \sqrt{xy}\,dy\,dx = \int_0^1 \sqrt{x}\cdot\left[\frac{2}{3}y^{\frac{3}{2}}\right]_{x^2}^{x} dx$$



$$= \frac{2}{3}\cdot\int_0^1 \sqrt{x}\left(x^{\frac{3}{2}} - x^3\right)dx = \frac{2}{3}\int_0^1 \left(x^2 - x^{\frac{7}{2}}\right)dx$$

$$= \frac{2}{3}\cdot\left[\frac{x^3}{3} - \frac{2}{9}x^{\frac{9}{2}}\right]_0^1 = \frac{2}{3}\cdot\underbrace{\left(\frac{1}{3}-\frac{2}{9}\right)}_{\frac{1}{9}} = \underline{\underline{\frac{2}{27}}}$$

4) $M: 4^x + 2 \leq y \leq 3 \cdot 2^x$

průsečíky křivek:

$$4^x + 2 = 3 \cdot 2^x$$

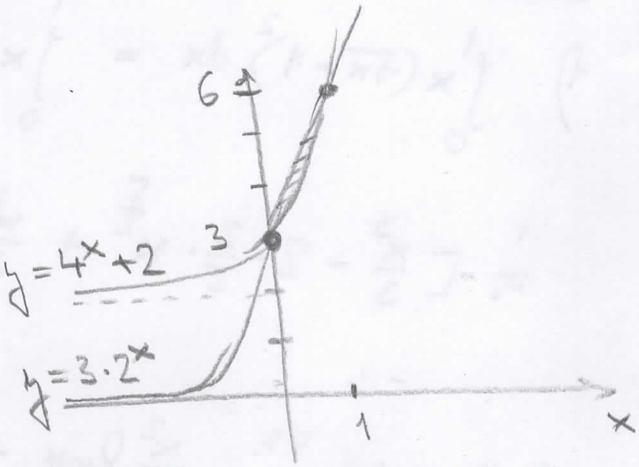
$$4^x - 3 \cdot 2^x + 2 = 0 \qquad 2^x = t$$

$$t^2 - 3 \cdot t + 2 = 0$$

$$(t-2)(t-1) = 0$$

$$t = 2 \ \vee \ t = 1$$

$$x = 1 \ \vee \ x = 0$$



$$\iint_M f(x,y)\,dy\,dx = \int_0^1 \int_{4^x+2}^{3\cdot 2^x} f(x,y)\,dy\,dx = \int_3^6 \int_{\log_2\left(\frac{y}{3}\right)}^{\log_4(y-2)} f(x,y)\,dx\,dy$$

5) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

$$\int_1^{\infty} \frac{x^2}{x^3+1}\,dx = \left| \begin{array}{l} x^3+1 = t \\ 3x^2\,dx = dt \\ x^2\,dx = \frac{1}{3}dt \end{array} \right| \frac{1}{3}\int_2^{\infty} \frac{1}{t}\,dt = \frac{1}{3}\left[\ln|t|\right]_2^{\infty}$$

$$= \frac{1}{3} \cdot \left(\underbrace{\lim_{t\to\infty} \ln|t|}_{+\infty} - \ln 2\right) = \underline{\underline{\infty}}$$

integrál $\int_1^{\infty} \frac{x^2}{x^3+1}\,dx$ diverguje, proto také řada diverguje

Funkce $f(x) := \frac{x^2}{x^3+1}$ splňuje:
- $f \geq 0$ na $\langle 1, \infty)$
- f je spojitá na $\langle 1, \infty)$
- $\lim_{x\to\infty} f(x) = 0$
- f je klesající od jistého x_0 $(\exists x_0 \in \mathbb{R}: f'(x) < 0$ na $\langle x_0, \infty))$

6) $8 + 6 + \frac{9}{2} + \frac{27}{8} + \frac{81}{32} + \dots = \frac{8}{1 - \frac{3}{4}} = \frac{8}{\frac{1}{4}} = \underline{\underline{32}}$