4. minitest - varianta A

Vázané extrémy - optimalizace s vazbou

27. 10. 2023

Najděte globální extrémy funkce

$$f(x, y) = y^2 - x^2 + 12x$$

s vazbovou podmínkou

$$x^2 + y^2 = 16.$$

vazba: $\underbrace{x^2 + y^2 - 16}_{=:\, g(x,y)} = 0$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} -2x + 12 & 2y \\ 2x & 2y \end{vmatrix} = (-2x+12)\cdot 2y - 2x \cdot 2y =$$

$$= 2y \cdot (-2x + 12 - 2x) = 2y \cdot (12 - 4x) = 0$$

$$\iff y = 0 \lor x = 3$$

I. $y = 0$: $x^2 - 16 = 0$

$x = \pm 4$

II. $x = 3$: $9 + y^2 - 16 = 0$

$y^2 = 7$

$y = \pm\sqrt{7}$

$f(4, 0) = -16 + 48 = 32$

$f(-4, 0) = -16 - 48 = -64$

$f(3, \pm\sqrt{7}) = 7 - 9 + 36 = 34$

minimum: $[-4, 0]$

maxima: $[3, \pm\sqrt{7}]$

4. minitest - varianta B

Vázané extrémy - optimalizace s vazbou

27. 10. 2023

Najděte globální extrémy funkce

$$f(x,y) = -x^2 + y^2 - 12y$$

s vazbovou podmínkou

$$x^2 + y^2 = 16.$$

$$g(x,y) := x^2 + y^2 - 16 = 0$$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} -2x & 2y-12 \\ 2x & 2y \end{vmatrix} = -4xy - 2x(2y-12)$$

$$= 2x \cdot (-2y - 2y + 12) = 2x(12 - 4y) = 0$$

$$\Leftrightarrow \quad x = 0 \lor y = 3$$

I. $\underline{x = 0}$: $y^2 = 16$

$y = \pm 4$

II. $\underline{y = 3}$: $x^2 + 9 = 16$

$x^2 = 7$

$x = \pm\sqrt{7}$

$f(0,4) = 16 - 48 = -32$

$f(0,-4) = 16 + 48 = 64$

$f(\pm\sqrt{7},3) = -7 + 9 - 36 = -34$

minima: $[\pm\sqrt{7}, 3]$

maximum: $[0, -4]$