

Funkce dvou proměnných

1. U funkcí $f(x, y)$ určete

$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y^2}, \quad \frac{\partial^2 f}{\partial x \partial y}:$$

a) $f(x, y) = x^3 + 2x^2y^2 - 3xy^3 + y^4 + 2$

b) $f(x, y) = (x^2 + 2) \cdot \sin(x + 3y)$

2. a) Pro $f(x, y) = \cos^2 2x \cdot \sin 3y$ určete $\frac{\partial^2 f}{\partial x \partial y}$ v bodě $\left[\frac{\pi}{8}, 0\right]$

b) Pro $f(x, y) = \arctg \frac{1+x}{1+y}$ určete $\frac{\partial^2 f}{\partial x^2}$ v bodě $[0, 1]$

c) Pro $f(x, y) = \frac{x^2y + 2y^2}{x-y}$ určete $\frac{\partial^2 f}{\partial y^2}$ v bodě $[1, 3]$

d) Pro $f(x, y) = \sqrt{x + \frac{1}{y}}$ určete $\frac{\partial^2 f}{\partial x \partial y}$ v bodě $\left[5, \frac{1}{4}\right]$

3. Určete obecnou rovnici tečné roviny v bodě T :

a) $f(x, y) = xy^2, \quad T_0 = [-2, 1] \in D_f$

b) $f(x, y) = x^2 + y^2, \quad T_0 = [0, 0] \in D_f$

c) $f(x, y) = x \cdot e^{x^2y}$ [$f(x, y) = x \cdot \exp(x^2y)$], $T_0 = [-1, 0] \in D_f$

d) $f(x, y) = x^3 + 2x^2y + xy^2 - y + 1, \quad T_0 = [1, 0] \in D_f$

4. Vypočtěte $\overrightarrow{\text{grad}} f(x, y)$ v bodě:

a) $f(x, y) = \frac{x}{y-x}, \quad A = [1, 2]$

b) $f(x, y) = x^3y^2 - y + 1, \quad B = [1, 1]$

c) $f(x, y) = \cos(\pi - x) \cdot (y + 1), \quad C = \left[\frac{\pi}{2}, 0\right]$

5. Určete lokální extrémů funkce:

a) $f(x, y) = x^2 + y^2 + 5$

b) $f(x, y) = 5x^2 + 3xy + y^2 + 4x - y + 6$

c) $f(x, y) = x^4 + y^4 + 2x^2y^2 - 4x$

Výsledky:

$$1. a) \frac{\partial^2 f}{\partial x^2} = 6x + 4y^2, \quad \frac{\partial^2 f}{\partial y^2} = 4x^2 - 18xy + 12y^2,$$

$$\frac{\partial^2 f}{\partial x \partial y} = 8xy - 9y^2$$

$$b) \frac{\partial^2 f}{\partial x^2} = 4x \cdot \cos(x + 3y) - x^2 \sin(x + 3y),$$

$$\frac{\partial^2 f}{\partial y^2} = -9(x^2 + 2) \sin(x + 3y),$$

$$\frac{\partial^2 f}{\partial x \partial y} = 6x \cos(x + 3y) - 3(x^2 + 2) \cdot \sin(x + 3y)$$

$$2. a) -6, \quad b) -\frac{4}{25}, \quad c) -\frac{3}{4}, \quad d) \frac{4}{27}$$

$$3. a) x - 4y - z + 4 = 0, \quad b) z = 0$$
$$c) x - y - z = 0 \quad d) 3x + y - z - 1 = 0$$

$$4. a) \overrightarrow{\text{grad}} f(A) = (2, -1) \quad b) \overrightarrow{\text{grad}} f(B) = (3, 1)$$
$$c) \overrightarrow{\text{grad}} f(C) = \left(1, -\frac{\pi}{2}\right)$$

5. a) $A = [0, 0]$ lokální minimum
b) $B = [-1, 2]$ lokální minimum
c) $C = [1, 0]$ lokální minimum