1.
$$f(x) = \frac{3x-1}{x^2-2x+1} = \frac{3x-1}{(x-1)^2} \qquad D_f = \mathbb{R} \setminus \{1\}$$

$$f'(x) = \frac{3(x-1)^2 - (3x-1)\cdot 2(x-1)}{(x-1)^4} = \frac{\cancel{(x-1)}\left(3x-3-6x+2\right)}{(x-1)^{\cancel{4}\,3}} =$$

$$= \frac{(-3x-1)}{(x-1)^3} = 0 \iff x = -\frac{1}{3}$$

$$\searrow \qquad \nearrow \qquad \searrow$$

$$f'<0 \quad -\frac{1}{3} \quad f'>0 \quad 1 \quad f'<0$$

$$f\left(-\frac{1}{3}\right) = \frac{-2}{\left(-\frac{4}{3}\right)^2} = -\frac{2}{\frac{16}{9}} = -\frac{9}{8}$$

lok. minimum: $\left[-\frac{1}{3}; -\frac{9}{8}\right]$

$$f''(x) = \left(-\frac{3x+1}{(x-1)^3}\right)' = -\frac{3(x-1)^3 - (3x+1)\,3(x-1)^2}{(x-1)^6} =$$

$$= -\frac{\cancel{(x-1)^2}\cdot\left(3(x-1) - 9x - 3\right)}{(x-1)^{\cancel{6}\,4}} = -\frac{-6x-6}{(x-1)^4} = \frac{6x+6}{(x-1)^4}$$

$$f''(x) = 0 \iff x = -1$$

$$f''<0 \quad -1 \quad f''>0 \quad 1 \quad f''>0$$

$$f(-1) = \frac{-4}{(-2)^2} = -1$$

inflexní bod: $[-1,-1]$

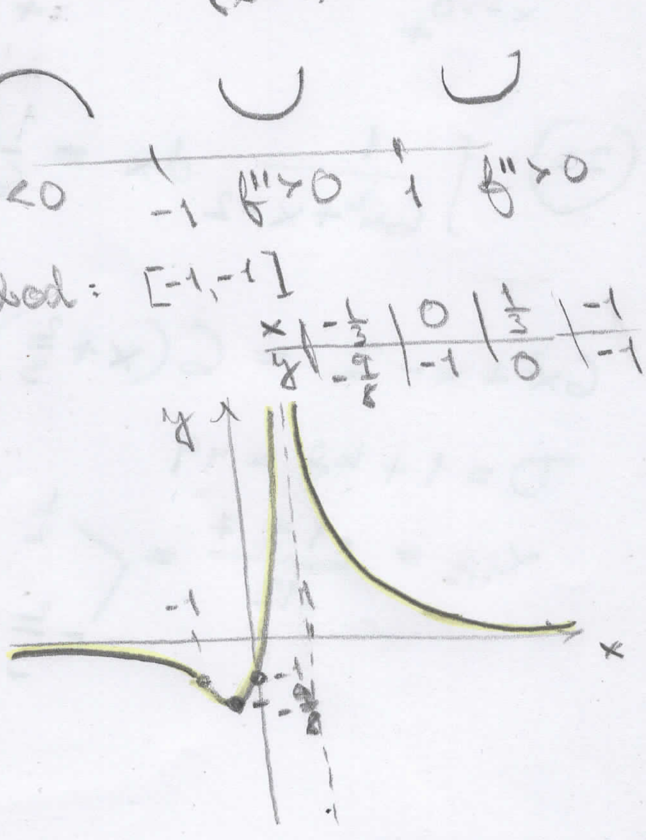
x	$-\frac{1}{3}$	0	$\frac{1}{3}$	-1
y	$-\frac{9}{8}$	-1	0	-1

$$\lim_{x\to\pm\infty} \frac{3x-1}{(x-1)^2} \overset{L.P.}{\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim_{x\to\pm\infty} \frac{3}{2(x-1)} = \underline{\underline{0}}$$

$$\lim_{x\to 1} \frac{3x-1}{(x-1)^2} = \text{"}\frac{2}{0_+}\text{"} = \underline{\underline{+\infty}}$$

průsečíky s osami: $[0,-1] = P_y$
$\left[\frac{1}{3}, 0\right] = P_x$

obor hodnot: $H_f = \left\langle -\frac{9}{8}, +\infty\right)$



2a

$$\int_4^7 x\sqrt{x-3}\,dx = \int_1^4 (t+3)\sqrt{t}\,dt =$$

$$\left|\begin{array}{l} x-3=t \\ dx=dt \end{array}\right| \Rightarrow x=t+3$$

$$= \int_1^4 \left(t^{\frac{3}{2}} + 3t^{\frac{1}{2}}\right) dt = \left[\frac{2}{5}t^{\frac{5}{2}} + 3\cdot\frac{2}{3}t^{\frac{3}{2}}\right]_1^4 =$$

$$= \frac{2}{5}\cdot 4^{\frac{5}{2}} + 2\cdot 4^{\frac{3}{2}} - \left(\frac{2}{5}+2\right) = \frac{64}{5} + 16 - \frac{2}{5} - 2 =$$

$$= \frac{62}{5} + 14 = \underline{\underline{\frac{132}{5}}}$$

2b

$$\int_0^1 \ln x\,dx \overset{P.P.}{=} \left[x\ln x\right]_0^1 - \int_0^1 1\,dx =$$

$$\left|\begin{array}{ll} u=\ln x & v'=1 \\ u'=\frac{1}{x} & v=x \end{array}\right|$$

$$= 1\ln 1 - \lim_{x\to 0_+} x\ln x - \left[x\right]_0^1 = 0-0-1 = \underline{\underline{-1}}$$

$$\lim_{x\to 0_+} x\ln x = \lim_{x\to 0_+} \frac{\ln x}{\frac{1}{x}} \overset{L.P.}{=} \lim_{x\to 0_+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0_+}(-x) = 0$$

2c

$$\int \frac{1}{6x^2+x-2}\,dx = \frac{1}{6}\left(\int \frac{-\frac{6}{7}}{x+\frac{2}{3}}\,dx + \int \frac{\frac{6}{7}}{x-\frac{1}{2}}\,dx\right) =$$

$$= \frac{1}{7}\left(-\ln\left|x+\tfrac{2}{3}\right| + \ln\left|x-\tfrac{1}{2}\right|\right)$$

$$= \underline{\underline{\frac{1}{7}\ln\frac{\left|x-\frac{1}{2}\right|}{\left|x+\frac{2}{3}\right|} + C}}$$

$$6x^2+x-2 = 6\left(x+\frac{2}{3}\right)\left(x-\frac{1}{2}\right)$$

$$D = 1+48 = 49$$

$$x_{1,2} = \frac{-1\pm 7}{12} = \begin{cases} \frac{1}{2} \\ -\frac{2}{3} \end{cases}$$

$$\frac{1}{\left(x+\frac{2}{3}\right)\left(x-\frac{1}{2}\right)} = \frac{A}{x+\frac{2}{3}} + \frac{B}{x-\frac{1}{2}}$$

$$1 = A\left(x-\frac{1}{2}\right) + B\left(x+\frac{2}{3}\right)$$

$$x=\frac{1}{2}:\ 1 = B\cdot\frac{7}{6} \qquad \boxed{B=\frac{6}{7}}$$

$$x=-\frac{2}{3}:\ 1 = A\cdot\left(-\frac{7}{6}\right) \qquad \boxed{A=-\frac{6}{7}}$$

(2d) $\int_0^1 \frac{x-1}{\sqrt{x}-1}\,dx \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \int_0^1 \frac{x-1}{x-1}\cdot(\sqrt{x}+1)\,dx = \int_0^1 (\sqrt{x}+1)\,dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} + x\right]_0^1 = \frac{2}{3}+1 = \underline{\underline{\frac{5}{3}}}$

(3.) $f(x,y) = y^3 \sin(xy)$

$z_0 = f(\frac{\pi}{2},1) = 1$

$\frac{\partial f}{\partial x} = y^4\cos(xy)\Big|_{[\frac{\pi}{2},1]} = 0$

$\frac{\partial f}{\partial y} = 3y^2\sin(xy) + y^3\cdot x\cos(xy)\Big|_{[\frac{\pi}{2},1]} = 3$

tečná rovina: $\boxed{z-1 = 3\cdot(y-1)}$

(4.) $f(x,y) = x^3+12x^2+y^2-6xy+3x-4y$ $\qquad D_f = \mathbb{R}^2$

$\frac{\partial f}{\partial x} = 3x^2+24x-6y+3 = 0 \quad |:3$

$\frac{\partial f}{\partial y} = 2y-6x-4 = 0 \quad |:2$

$x^2+8x-2y+1 = 0$

$y-3x-2 = 0 \implies \boxed{y = 3x+2}$

$x^2+8x-2(3x+2)+1 = 0$

$x^2+2x-3 = 0$

$(x+3)(x-1) = 0$

$\underline{x=-3} \quad \vee \quad \underline{x=1}$

$y=-7 \qquad y=5$

stacionární body: [-3;-7], [1,5]

$$\frac{\partial^2 f}{\partial x^2} = 6x + 24$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -6$$

HESSOVA MATICE

$$H(x,y) = \begin{pmatrix} 6x+24 & -6 \\ -6 & 2 \end{pmatrix}$$

$$H(-3,-7) = \begin{pmatrix} +6 & -6 \\ -6 & 2 \end{pmatrix} \quad \begin{vmatrix} 6 & -6 \\ -6 & 2 \end{vmatrix} = 12 - 36 < 0$$

$\Rightarrow$ [-3,-7] je sedlový bod

$$H(1,5) = \begin{pmatrix} 30 & -6 \\ -6 & 2 \end{pmatrix} \quad \begin{vmatrix} 30 & -6 \\ -6 & 2 \end{vmatrix} = 60 - 36 = 24 > 0$$

$$\wedge \ \frac{\partial^2 f}{\partial x^2}(1,5) > 0$$

$\Rightarrow$ [1,5] je lok. minimum

$$f(1,5) = 1 + 12 + 25 - 30 + 3 - 20 = -9$$