1. $f(x) = \dfrac{x^2 - x - 2}{x - 3}$ $D_f = \mathbb{R} \setminus \{3\}$

x	0	2	-1	1	5
y	$\frac{2}{3}$	0	0	1	9

průsečíky s osami:

$f(0) = \dfrac{2}{3}$ $P_y = [0; \frac{2}{3}]$

$f(x) = 0 \iff x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2 \lor x = -1$

$P_{x_1} = [2; 0]$
$P_{x_2} = [-1; 0]$

$$f'(x) = \frac{(2x-1)(x-3) - 1(x^2-x-2)}{(x-3)^2} = \frac{2x^2 - x - 6x + 3 - x^2 + x + 2}{(x-3)^2} =$$

$$= \frac{x^2 - 6x + 5}{(x-3)^2} = \frac{(x-5)(x-1)}{(x-3)^2}$$

$f' > 0$ 1 $f' < 0$ 3 $f' < 0$ 5 $f' > 0$

$f'(x) = 0 \iff x = 1 \lor x = 5$

$f(1) = \dfrac{1^2 - 1 - 2}{1 - 3} = 1$ lok. maximum: $[1; 1]$

$f(5) = \dfrac{5^2 - 5 - 2}{5 - 3} = \dfrac{18}{2} = 9$ lok. minimum: $[5; 9]$

$$f''(x) = \frac{(2x-6)(x-3)^2 - 2(x-3)(x^2-6x+5)}{(x-3)^4} = \frac{\cancel{(x-3)}\left((2x-6)(x-3) - 2(x^2-6x+5)\right)}{(x-3)^{\cancel{4}\,3}}$$

$$= \frac{2x^2 - 6x - 6x + 18 - 2x^2 + 12x - 10}{(x-3)^3} = \frac{8}{(x-3)^3}$$

$f'' < 0$ 3 $f'' > 0$

$$\left.\begin{aligned} \lim_{x \to 3_+} \frac{x^2 - x - 2}{x - 3} &= \frac{4}{0_+} = +\infty \\ \lim_{x \to 3_-} \frac{x^2 - x - 2}{x - 3} &= \frac{4}{0_-} = -\infty \end{aligned}\right\} \text{svislá asymptota } x = 3$$

šikmá asymptota: $y = kx + q$

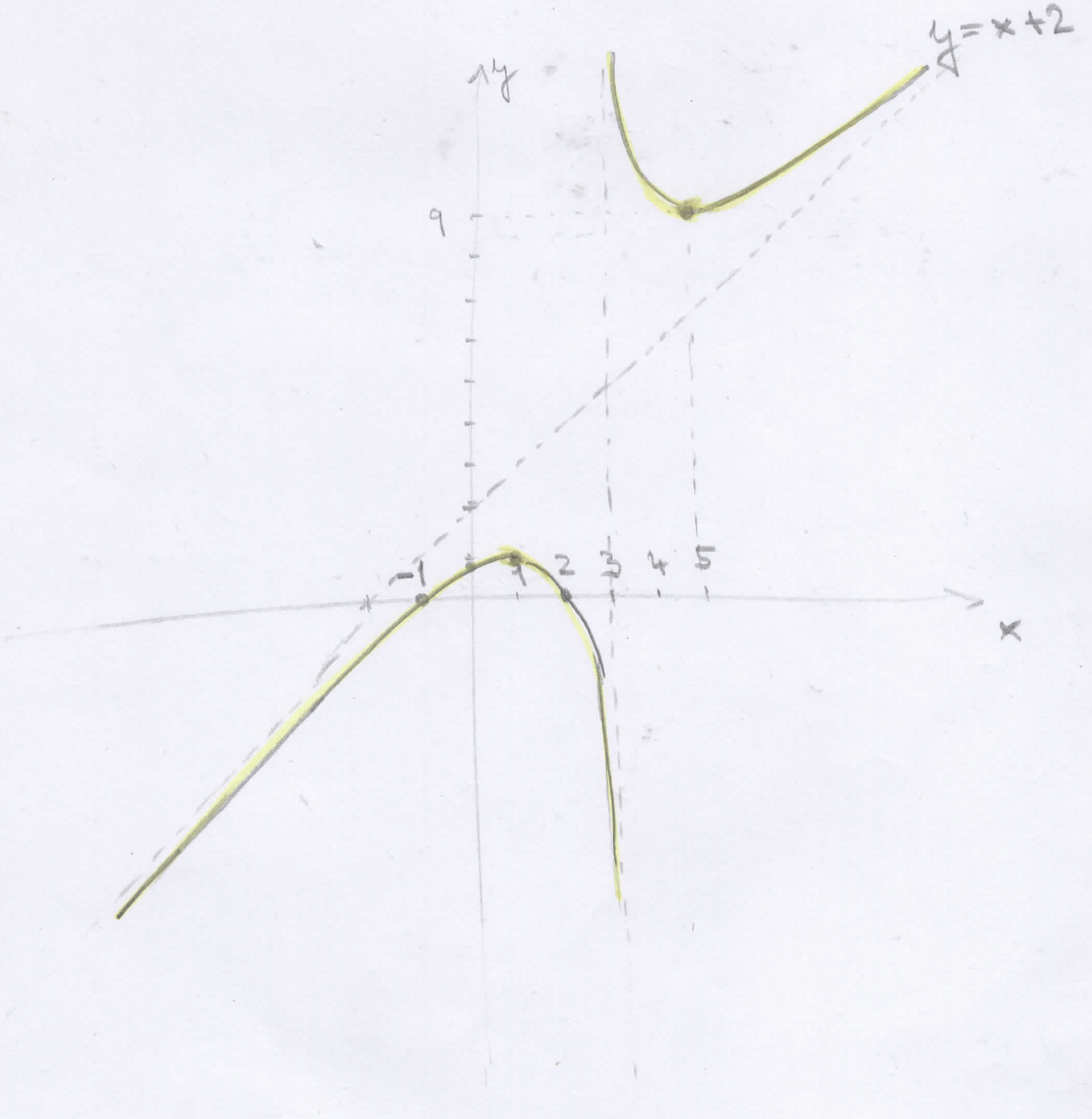
$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{x^2 - x - 2}{x^2 - 3x} \overset{\text{L.P.}}{\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim \frac{2x - 1}{2x - 3} \overset{\text{L.P.}}{\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim \frac{2}{2} = 1$$

$$q = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} \left(\frac{x^2 - x - 2}{x - 3} - 1x\right) = \lim_{x \to \pm\infty} \frac{\cancel{x^2} - x - 2 - \cancel{x^2} + 3x}{x - 3}$$

$$= \lim \frac{2x - 2}{x - 3} \overset{\text{L.P.}}{=} \lim \frac{2}{1} = 2$$

šikmá asymptota: $y = x + 2$

Obor hodnot $H_f = (-\infty, 1\rangle \cup \langle 9, \infty)$



(2a) $$\int_{-1}^{1} \frac{x^5}{x^6+1}\,dx = \frac{1}{6}\int_{2}^{2}\frac{dt}{t} = \frac{1}{6}\cdot\left[\ln|t|\right]_{2}^{2} = \underline{\underline{0}}$$

$$\left| \begin{array}{l} x^6+1 = t \\ 6x^5\,dx = dt \end{array} \right|$$

(2b) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} x\sin 3x\,dx \stackrel{\text{P.P.}}{=} \left[-\frac{x}{3}\cos 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \frac{1}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos 3x\,dx =$$

$$\left| \begin{array}{ll} u = x & v' = \sin 3x \\ u' = 1 & v = -\frac{1}{3}\cos 3x \end{array} \right|$$

$$= -\frac{\pi}{9}\underbrace{\cos\pi}_{=-1} + \frac{\pi}{18}\underbrace{\cos\frac{\pi}{2}}_{=0} + \frac{1}{9}\underbrace{\left[\sin 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}}_{\underbrace{\sin\pi}_{0} - \underbrace{\sin\frac{\pi}{2}}_{1}} =$$

$$= \underline{\underline{\frac{\pi}{9} - \frac{1}{9}}}$$

(2c) $$\int\frac{1}{x^3+x}\,dx \stackrel{(*)}{=} \int\frac{1}{x}\,dx - \int\frac{x}{x^2+1}\,dx = \underline{\underline{\ln|x| - \frac{1}{2}\ln|x^2+1| + c}}$$

$$\left| \begin{array}{l} x^2+1 = t \\ 2x\,dx = dt \end{array} \right|$$

(*) $$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \qquad \Big| \cdot x(x^2+1)$$

$$1 = A(x^2+1) + (Bx+C)x$$

$x=0$: $\boxed{1 = A}$

$$1 = x^2 + 1 + Bx^2 + Cx$$

I. $\boxed{C = 0}$

II. $0 = 1 + B$

$\boxed{B = -1}$

(2d) $$\int_0^1 x\left(1 - 2\sqrt[3]{x}\right)^2 dx = \int_0^1 x\left(1 - 4x^{\frac{1}{3}} + 4x^{\frac{2}{3}}\right)dx$$

$$= \int_0^1\left(x - 4x^{\frac{4}{3}} + 4x^{\frac{5}{3}}\right)dx = \left[\frac{x^2}{2} - 4\,\frac{x^{\frac{7}{3}}}{\frac{7}{3}} + 4\cdot\frac{x^{\frac{8}{3}}}{\frac{8}{3}}\right]_0^1 =$$

$$= \frac{1}{2} - 4\cdot\frac{3}{7} + 4\cdot\frac{3}{8} = \frac{1}{2} - \frac{12}{7} + \frac{3}{2} = \frac{14-12}{7} = \underline{\underline{\frac{2}{7}}}$$

(3.) $f(x,y) = x e^{xy}$

$f(3,0) = 3$

$$\frac{\partial f}{\partial x} = e^{xy} + xy e^{xy} \Big|_{[3,0]} = 1$$

$$\frac{\partial f}{\partial y} = x^2 e^{xy} \Big|_{[3,0]} = 9$$

tečná rovina: $z - 3 = 1(x-3) + 9y$

$$\boxed{x + 9y - z = 0}$$

(4.) $f(x,y) = x^3 + y^2 + 15x^2 - 6xy + 18x - 6y + 5$

$$\frac{\partial f}{\partial x} = 3x^2 + 30x - 6y + 18 = 0 \quad |:3$$

$$\frac{\partial f}{\partial y} = 2y - 6x - 6 = 0 \quad |:2$$

$x^2 + 10x - 2y + 6 = 0$ ← dosazení

$y - 3x - 3 = 0 \Rightarrow \boxed{y = 3x + 3}$

$x^2 + 10x - 2\cdot(3x+3) + 6 = 0$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \lor x = -4$

$y = 3 \quad y = -9$

$$\frac{\partial^2 f}{\partial x^2} = 6x + 30$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -6$$

Hessova matice: $H(x,y) = \begin{pmatrix} 6x+30 & -6 \\ -6 & 2 \end{pmatrix}$

$H(0,3) = \begin{pmatrix} 30 & -6 \\ -6 & 2 \end{pmatrix} \quad \begin{vmatrix} 30 & -6 \\ -6 & 2 \end{vmatrix} = 60 - 36 > 0$

$\land \ \frac{\partial^2 f}{\partial x^2}(0,3) > 0 \Rightarrow [0,3]$ je lok. min.

$H(-4,-9) = \begin{pmatrix} 6 & -6 \\ -6 & 2 \end{pmatrix} \quad \begin{vmatrix} 6 & -6 \\ -6 & 2 \end{vmatrix} = 12 - 36 < 0$

$\Rightarrow [-4,-9]$ je sedlo