1. $f(x)=\frac{1-x^2}{1+x^2}$ $\quad D_f=\mathbb{R}$

x	0	$\pm\frac{\sqrt{3}}{3}$	± 1
y	1	$\frac{1}{2}$	0

$$f'(x)=\frac{(-2x)(1+x^2)-2x(1-x^2)}{(1+x^2)^2}=\frac{-4x}{(1+x^2)^2}=0 \iff x=0$$

lok. maximum: $[0,1]$

$$\nearrow \quad \searrow$$

$f'>0 \quad 0 \quad f'<0$

$$f''(x)=-\frac{4(1+x^2)^2-4x\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4}=-\frac{\cancel{(1+x^2)}\left(4(1+x^2)-16x^2\right)}{(1+x^2)^{\cancel{4}\,3}}=$$

$$=-\frac{4-12x^2}{(1+x^2)^3}=\frac{12x^2-4}{(1+x^2)^3}=0 \iff x=\pm\frac{\sqrt{3}}{3}$$

$$f\left(\pm\frac{\sqrt{3}}{3}\right)=\frac{1-\frac{1}{3}}{1+\frac{1}{3}}=\frac{\frac{2}{3}}{\frac{4}{3}}=\frac{2}{4}=\frac{1}{2}$$

$$\cup \quad \cap \quad \cup$$

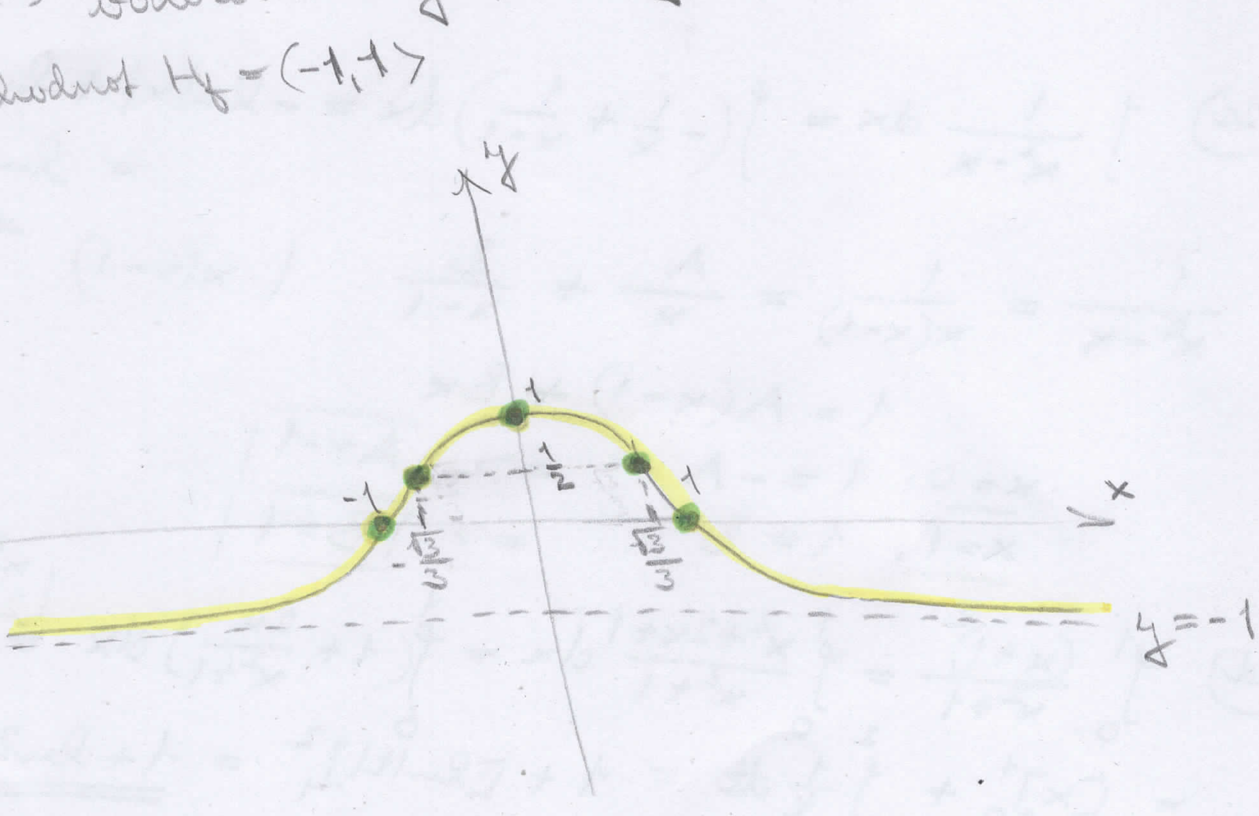
$f''>0 \quad -\frac{\sqrt{3}}{3} \quad f''<0 \quad \frac{\sqrt{3}}{3} \quad f''>0$

inflexní body: $\left[\pm\frac{\sqrt{3}}{3};\ \frac{1}{2}\right]$

$$\lim_{x\to\pm\infty}\frac{1-x^2}{1+x^2}\ \overset{\text{L.P.}}{\underset{„\frac{\infty}{\infty}“}{=}}\ \lim_{x\to\pm\infty}\frac{-2x}{2x}=-1$$

$\Rightarrow$ vodorovná asymptota $y=-1$

Obor hodnot $H_f=(-1,1\rangle$



(2a) $\int_{-\frac{1}{2}}^{0} (2x+1)^{49}\,dx = \frac{1}{2}\int_0^1 t^{49}\,dt = \frac{1}{2}\cdot\underbrace{\left[\frac{t^{50}}{50}\right]_0^1}_{\frac{1}{50}} = \underline{\underline{\frac{1}{100}}}$

$$\left|\begin{aligned} 2x+1 &= t \\ 2dx &= dt \\ dx &= \tfrac{1}{2}dt \end{aligned}\right|$$

(2b) $\int_0^1 x\ln x\,dx \overset{P.P.}{=} \left[\frac{x^2}{2}\ln x\right]_0^1 - \int_0^1 \frac{x}{2}\,dx =$

$$\left|\begin{aligned} u &= \ln x & v' &= x \\ u' &= \tfrac{1}{x} & v &= \tfrac{x^2}{2} \end{aligned}\right|$$

$= \underbrace{\frac{1^2}{2}\ln 1}_{0} - \underbrace{\lim_{x\to 0_+}\frac{x^2}{2}\ln x}_{0^{(*)}} - \underbrace{\frac{1}{2}\cdot\left[\frac{x^2}{2}\right]_0^1}_{\frac{1}{2}} = \underline{\underline{-\frac{1}{4}}}$

$=$

(*) $\lim_{x\to 0_+}\frac{x^2}{2}\ln x = \frac{1}{2}\lim_{x\to 0_+}\frac{\ln x}{\frac{1}{x^2}} \overset{L.P.}{=} \frac{1}{2}\lim_{x\to 0_+}\frac{\frac{1}{x}}{-\frac{2}{x^3}} =$

$= \frac{1}{2}\lim_{x\to 0_+}\left(-\frac{x^3}{2x}\right) = \frac{1}{2}\lim_{x\to 0_+}\left(-\frac{x^2}{2}\right) = 0$

(2c) $\int \frac{1}{x^2-x}\,dx = \int\left(-\frac{1}{x}+\frac{1}{x-1}\right)dx = -\ln|x| + \ln|x-1| + c$
$= \underline{\underline{\ln\frac{|x-1|}{|x|} + c}}$

$\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \quad \Big|\cdot x(x-1)$

$1 = A(x-1) + Bx$

$\underline{x=0}: \ 1 = -A \quad \Rightarrow \boxed{A=-1}$

$\underline{x=1}: \ 1 = B \quad \Rightarrow \boxed{B=1}$

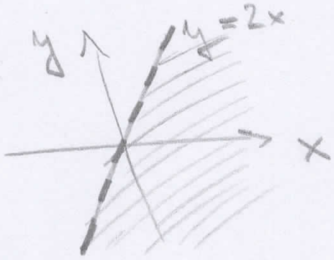
(2d) $\int_0^1 \frac{(x+1)^2}{x^2+1} = \int_0^1 \frac{x^2+2x+1}{x^2+1}\,dx = \int_0^1\left(1+\frac{2x}{x^2+1}\right)dx = \quad \left|\begin{aligned} x^2+1 &= t \\ 2x\,dx &= dt \end{aligned}\right|$

$= [x]_0^1 + \int_1^2 \frac{1}{t}\,dt = 1 + [\ln|t|]_1^2 = \underline{\underline{1+\ln 2}}$

3. $f(x,y) = \ln(2x-y)$

$2x - y > 0$

$y < 2x$



$D_f = \{(x,y) \in \mathbb{R}^2 ;\ y < 2x\}$

$f(1,1) = \ln 1 = 0$

$\frac{\partial f}{\partial x} = 2 \cdot \frac{1}{2x-y} \Big|_{[1,1]} = 2$

$\frac{\partial f}{\partial y} = -\frac{1}{2x-y} \Big|_{[1,1]} = -1$

tečná rovina: $z - 0 = 2(x-1) + (-1)(y-1)$

$z = 2x - y - 1$

4. $f(x,y) = 6xy - \frac{y}{x} + e^y$

$\frac{\partial f}{\partial x} = 6y + \frac{y}{x^2} = y\left(6 + \frac{1}{x^2}\right) = 0 \iff \boxed{y = 0}$

$\frac{\partial f}{\partial y} = 6x - \frac{1}{x} + e^y = 0 \longleftarrow$ dosazení

$\frac{\partial^2 f}{\partial x^2} = -\frac{2y}{x^3}$

$\frac{\partial^2 f}{\partial y^2} = e^y$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 6 + \frac{1}{x^2}$

$6x - \frac{1}{x} + 1 = 0 \quad | \cdot x$

$6x^2 - 1 + x = 0$

$6x^2 + x - 1 = 0$

$D = 1 + 24 = 25$

$x_{1,2} = \frac{-1 \pm 5}{12} = \begin{cases} -\frac{1}{2} \\ \frac{1}{3} \end{cases}$

stacionární body: $[-\frac{1}{2}; 0]$, $[\frac{1}{3}; 0]$

$\left|H(-\tfrac{1}{2}, 0)\right| = \begin{vmatrix} 0 & 10 \\ 10 & 1 \end{vmatrix} = -100 < 0$

$\Rightarrow [-\frac{1}{2}; 0]$ je sedlo

$\left|H(\tfrac{1}{3}, 0)\right| = \begin{vmatrix} 0 & 15 \\ 15 & 1 \end{vmatrix} = -225 < 0$

$\Rightarrow [\frac{1}{3}; 0]$ je sedlo