1.

$$A = \begin{pmatrix} 1 & 2 & a \\ 2 & a & 4 \\ 1 & a & 5 \end{pmatrix}$$

$$\det A = 5a + 2a^2 + 8 - (a^2 + 4a + 20) = a^2 + a - 12$$

$$\det A = 0 \iff a^2 + a - 12 = 0$$
$$(a+4)(a-3) = 0$$
$$a = -4 \lor a = 3$$

$\forall a \in \mathbb{R} \setminus \{3; -4\}: \ h(A) = 3$

$\underline{a = 3}$: $$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$h(A) = 2$

$\underline{a = -4}$: $$A = \begin{pmatrix} 1 & 2 & -4 \\ 2 & -4 & 4 \\ 1 & -4 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -4 \\ 0 & -8 & 12 \\ 0 & -6 & 9 \end{pmatrix} \begin{matrix} \\ \vert :4 \\ \vert :3 \end{matrix} \sim \begin{pmatrix} 1 & 2 & -4 \\ 0 & -2 & 3 \\ \cancel{0} & \cancel{-2} & \cancel{3} \end{pmatrix}$$

$h(A) = 2$

2.

$$\begin{vmatrix} 2 & 4 & 9 & 0 & 5 \\ 0 & 1 & 2 & 0 & 3 \\ 1 & 4 & 5 & 2 & 6 \\ 0 & 4 & 3 & 0 & 5 \\ 0 & 2 & 0 & 0 & 0 \end{vmatrix} = 2 \cdot (-1)^{2+5} \cdot \begin{vmatrix} 2 & 9 & 0 & 5 \\ 0 & 2 & 0 & 3 \\ 1 & 5 & 2 & 6 \\ 0 & 3 & 0 & 5 \end{vmatrix} = (-2) \cdot 2 \cdot (-1)^{3+3} \cdot \begin{vmatrix} 2 & 9 & 5 \\ 0 & 2 & 3 \\ 0 & 3 & 5 \end{vmatrix}$$

$$= (-4) \cdot 2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = -8 \cdot (10 - 9) = \underline{\underline{-8}}$$

3.

$$f(x) = \ln(x^2 + 1) \qquad x^2 + 1 > 0 \ \forall x \in \mathbb{R} \qquad D_f = \mathbb{R}$$

$$f'(x) = \frac{2x}{x^2+1} = 0 \iff x = 0$$

$f' < 0$ ↘ 0 ↗ $f' > 0$

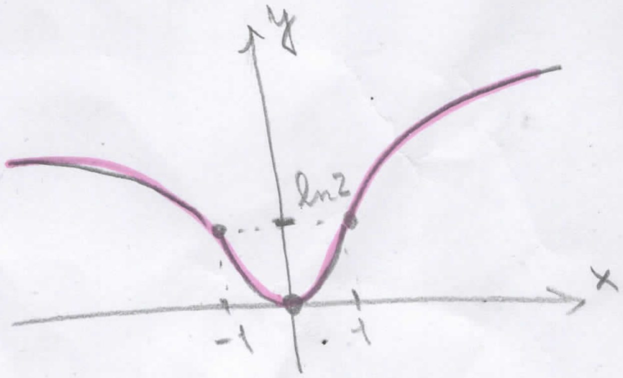
$$f''(x) = \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{-2x^2 + 2}{(x^2+1)^2}$$

$f'' < 0$ ∩ -1 ∪ $f'' > 0$ 1 ∩ $f'' < 0$

$$f''(x) = 0 \iff -2x^2 + 2 = 0 \quad \vert :(-2)$$
$$x^2 - 1 = 0$$
$$x = \pm 1$$

lok. minimum: $[0; 0]$

inflexní body: $[\pm 1, \ln 2]$

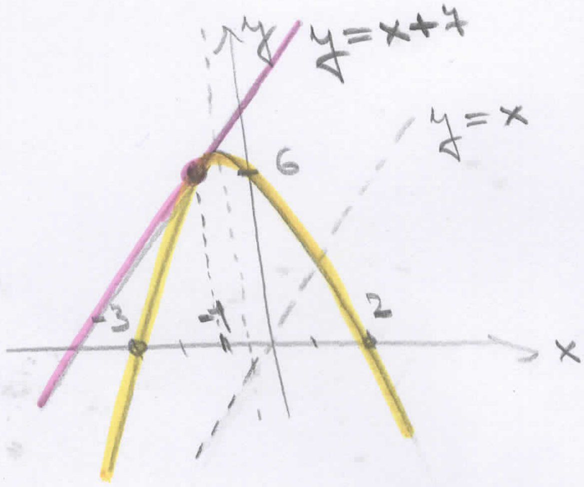


4. $f(x) = -x^2 - x + 6 \quad = -(x^2 + x - 6) = -(x+3)(x-2)$

$f'(x) = -2x - 1 \overset{?}{=} 1$

$-2x = 2$

$x = -1$



tečna: $t: y = x + c$

tečný bod $T = [-1, f(-1)] = [-1, 6]$

$T \in t \Rightarrow 6 = -1 + c$

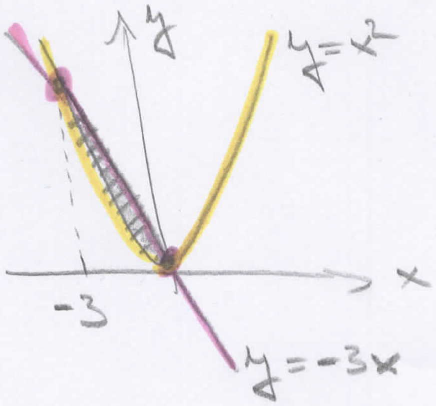
$c = 7$

tečna: $\boxed{y = x + 7}$

5. $$\int_0^1 \frac{1+5x}{\sqrt{x\sqrt{x}}}\,dx = \int_0^1 \frac{1+5x}{x^{\frac{3}{4}}}\,dx = \int_0^1 \left(x^{-\frac{3}{4}} + 5x^{\frac{1}{4}}\right) dx$$

$$= \left[4x^{\frac{1}{4}} + \cancel{5}\cdot\frac{4}{\cancel{5}}x^{\frac{5}{4}}\right]_0^1 = 4 + 4 = \underline{\underline{8}}$$

6.



průsečíky křivek:

$x^2 = -3x$

$x^2 + 3x = 0$

$x(x+3) = 0$

$\underline{x = 0} \vee \underline{x = -3}$

$$\int_{-3}^{0} (-3x)\,dx - \int_{-3}^{0} x^2\,dx = \left[-\frac{3x^2}{2}\right]_{-3}^{0} - \left[\frac{x^3}{3}\right]_{-3}^{0} =$$

$$= 0 + \frac{27}{2} - \left(0 + \frac{27}{3}\right) = 27\cdot\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{27}{6} = \underline{\underline{\frac{9}{2}\ [j^2]}}$$