1) $\exists a,b \in \mathbb{R}$ takové, že $a\cdot\begin{pmatrix}2\\3\\-2\end{pmatrix} + b\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$?

$$\left(\begin{array}{cc|c}2&2&1\\3&1&2\\-2&2&-2\end{array}\right)\oplus \sim \left(\begin{array}{cc|c}2&2&1\\3&1&2\\0&4&-1\end{array}\right)\begin{array}{l}|\cdot(-3)\\|\cdot 2\end{array}\oplus \sim \left(\begin{array}{cc|c}2&2&1\\0&-4&1\\0&4&-1\end{array}\right)$$

$\Rightarrow \boxed{b = -\frac{1}{4}}$

$$2a - 2\cdot\frac{1}{4} = 1$$
$$2a = \frac{3}{2}$$
$$\boxed{a = \frac{3}{4}}$$

ano, $\frac{3}{4}\cdot\vec{u_1} + \left(-\frac{1}{4}\right)\cdot\vec{u_2} = \vec{u_3}$.

$\Rightarrow$ vektory jsou LZ $\Rightarrow$ $\det A = 0$

matice je singulární

A^{-1} neexistuje

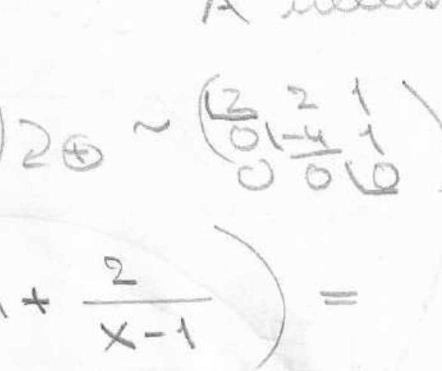
$$\begin{pmatrix}2&2&1\\3&1&2\\-2&2&-2\end{pmatrix} \sim \begin{pmatrix}2&2&1\\0&-4&1\\0&4&-1\end{pmatrix}\oplus \sim \begin{pmatrix}2&2&1\\0&-4&1\\0&0&0\end{pmatrix} \quad h(A) = 2$$

2) $$f(x) = \left(\frac{2x^3}{x-1} - x^2\right) : \left(1 + \frac{2}{x-1}\right) =$$

$$= \frac{2x^3 - x^2(x-1)}{x-1} : \frac{x-1+2}{x-1} = \frac{x^3+x^2}{\cancel{x-1}}\cdot\frac{\cancel{x-1}}{x+1} = \frac{x^2\cancel{(x+1)}}{\cancel{x+1}}$$
$$= x^2$$

$D_f = \mathbb{R} \setminus \{\pm 1\}$

$H_f = \langle 0,1) \cup (1,\infty)$



3) $f(x) = \frac{3x^2 - 8x + 4}{x^2} = 3 - \frac{8}{x} + \frac{4}{x^2}$ $D_f = \mathbb{R} \setminus \{0\}$

$f'(x) = \frac{8}{x^2} - \frac{8}{x^3} = \frac{8x - 8}{x^3} = 0 \iff x = 1$

$f''(x) = -\frac{16}{x^3} + \frac{24}{x^4} = \frac{-16x + 24}{x^4}$

$f' > 0$ 0 $f' < 0$ 1 $f' > 0$

$f''(x) = 0 \iff -16x + 24 = 0$

$x = \frac{24}{16} = \frac{3}{2}$

$f'' > 0$ 0 $f'' > 0$ $\frac{3}{2}$ $f'' < 0$

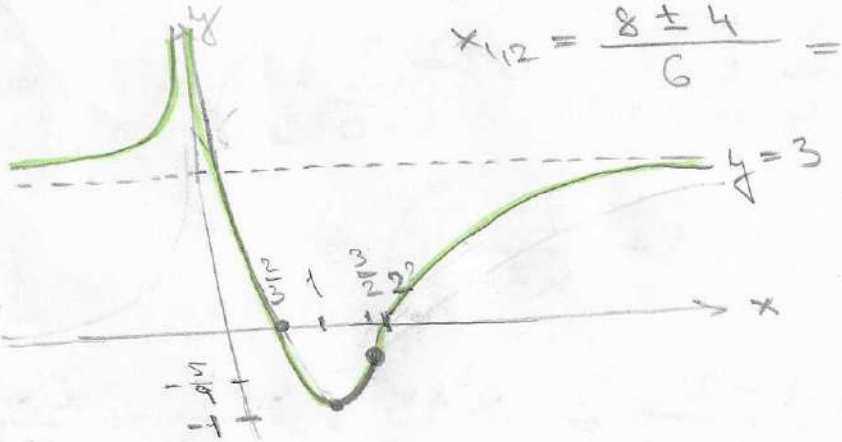
$f(1) = 3 - 8 + 4 = -1$

$f\left(\frac{3}{2}\right) = 3 - \frac{16}{3} + \frac{16}{9} = \frac{27 - 48 + 16}{9} = -\frac{5}{9}$

$f(x) = 0 \iff 3x^2 - 8x + 4 = 0$

$D = 64 - 4 \cdot 3 \cdot 4 = 16$

$x_{1,2} = \frac{8 \pm 4}{6} = \left\langle \begin{array}{l} 2 \\ \frac{2}{3} \end{array} \right.$



$H_f = \langle -1, \infty)$

$\lim_{x \to 0} f(x) = \frac{4}{0_+} = +\infty$

$\lim_{x \to \pm\infty} f(x) = 3$

5a) $\lim_{x \to \infty} \frac{2^x + 3^x}{4^x} = \lim_{x \to \infty} \left(\left(\frac{2}{4}\right)^x + \left(\frac{3}{4}\right)^x \right) = 0$

$\lim_{x \to 0} \frac{x^4 + 3x + 1 - e^{2x}}{x e^{-2x} - x + \sin(3x)} \overset{\text{L.P.}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x \to 0} \frac{4x^3 + 3 - 2e^{2x}}{e^{-2x} + x e^{-2x} \cdot (-2) - 1 + 3\cos(3x)} = \frac{4 \cdot 0^3 + 3 - 2e^0}{e^0 + 0 - 1 + 3\cos 0} = \frac{3 - 2}{1 - 1 + 3} = \underline{\underline{\frac{1}{3}}}$

6a) $\int_0^1 (2x-1)(\sqrt{x}+1)^2\,dx = \int_0^1 (2x-1)(x+2\sqrt{x}+1)\,dx =$

$= \int_0^1 \left(2x^2+4x^{\frac{3}{2}}+2x-x-2x^{\frac{1}{2}}-1\right)dx$

$= \left[2\frac{x^3}{3}+4\cdot\frac{2}{5}x^{\frac{5}{2}}+\frac{x^2}{2}-2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-x\right]_0^1$

$= \frac{2}{3}+\frac{8}{5}+\frac{1}{2}-\frac{4}{3}-1 = \frac{-20+48+15-30}{30} = \frac{13}{30}$

6b) $\int \operatorname{tg} x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{dt}{t} = -\ln|t|$ $\left|\begin{array}{l}\cos x = t\\ -\sin x\,dx = dt\end{array}\right|$

$= -\ln|\cos x| + c$

7) průsečíky hyperboly $y=\frac{2}{x}$ a přímky $y=3-x$:

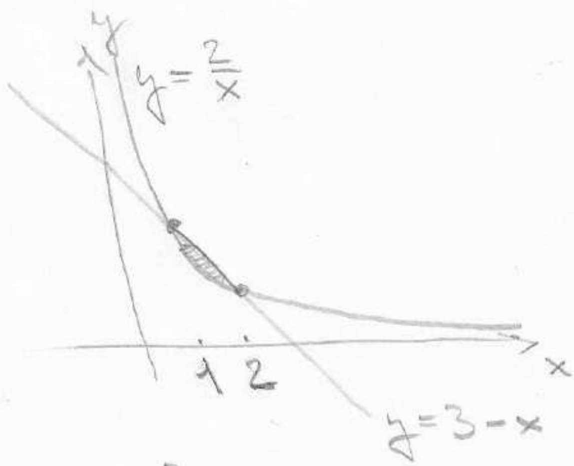
$\frac{2}{x} = 3-x \quad |\cdot x$

$2 = 3x - x^2$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x=2 \vee x=1$



$\underbrace{\int_1^2 (3-x)\,dx}_{\text{obsah pod přímkou}} - \underbrace{\int_1^2 \frac{2}{x}\,dx}_{\text{obsah pod hyperbolou}} = \left[3x-\frac{x^2}{2}-2\ln|x|\right]_1^2$

$= 6-2-2\ln 2 - (3-\frac{1}{2}-2\ln 1) =$

$= \frac{3}{2}-2\ln 2$