1.

$$A = \begin{pmatrix} a & 2a & 3a & 4a \\ 0 & 3a-a^2 & a^2-9 & 0 \\ 0 & 0 & a-3 & 0 \\ 0 & 0 & 0 & a^2-2a \end{pmatrix}$$

$$\det A = a\cdot(3a-a^2)(a-3)(a^2-2a) = a^3(3-a)(a-3)(a-2)$$

$$\det A = 0 \iff a=0 \ \vee\ a=2 \ \vee\ a=3$$

$\forall a \in \mathbb{R}\setminus\{0;2;3\}: \ h(A)=4$ (A je regulární)

<u>$a=0$</u>: $A = \begin{pmatrix} \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & -9 & 0 \\ 0 & 0 & -3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \begin{matrix} \\ \\ (-3) \\ \\ \end{matrix} \sim \begin{pmatrix} 0 & 0 & -3 & 0 \end{pmatrix}$ $h(A)=1$

<u>$a=2$</u>: $A = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 0 & 2 & -5 & 0 \\ 0 & 0 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$ $h(A)=3$

<u>$a=3$</u>: $A = \begin{pmatrix} 3 & 6 & 9 & 12 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & 0 & 3 \end{pmatrix}$ $h(A)=2$

2.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & -1 & 3 & 6 \\ 3 & 1 & k & 14 \end{array}\right) \begin{matrix} \\ (-1) \\ (-3) \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 2 & 2 \\ 0 & -2 & k-3 & 2 \end{array}\right) \begin{matrix} \\ \\ (-1) \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & k-5 & 0 \end{array}\right)$$

<u>I. $k \neq 5$</u> $\Rightarrow$ $z=0,\ y=-1,\ x=5$ $\Rightarrow$ protnou se v bodě

<u>II. $k=5$</u>: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & 1 & 1 \end{array}\right)$ $\Rightarrow$ protnou se v přímce

$$\left.\begin{aligned} x &= -2t+5 \\ y &= t-1 \\ z &= t \end{aligned}\right|, \ t \in \mathbb{R}$$

3.

$$f(x) = x^3 + 3x^2 = x^2(x+3) = 0 \iff x=0 \ \vee\ x=-3$$

$$f'(x) = 3x^2 + 6x = 3x(x+2) = 0 \iff x=0 \ \vee\ x=-2$$

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$f'>0$ $\quad -2 \quad$ $f'<0$ $\quad 0 \quad$ $f'>0$

$$f''(x) = 6x + 6 = 0 \iff x = -1$$

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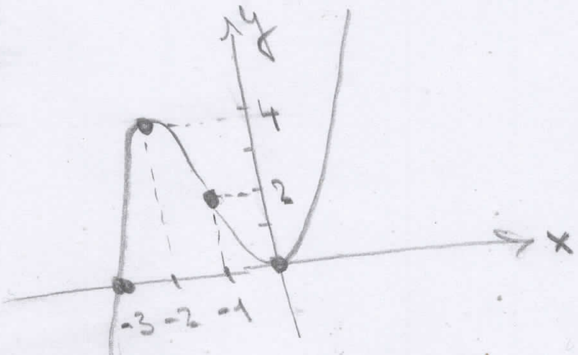
$f''<0$ $\quad -1 \quad$ $f''>0$

$D_f = \mathbb{R}$

$H_f = \mathbb{R}$



inflexní bod $[-1, 2]$

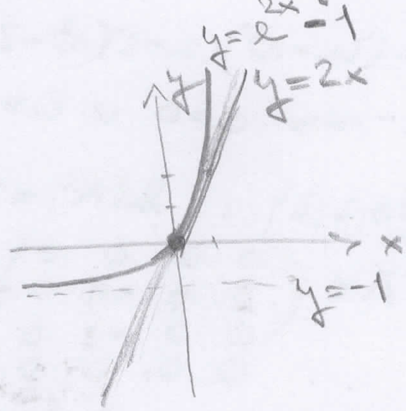
lok. max. $[-2, 4]$

lok. min. $[0, 0]$

(4.) $f(x) = e^{2x} - 1 = 0 \iff x = 0$

$f'(x) = 2e^{2x} \Big|_{x=0} = 2$

tečný bod: $[0,0]$



$t: \; y = 2x + q$

$[0,0] \in t \Rightarrow q = 0$

tečna: $\boxed{y = 2x}$

(5.) $$\int_0^{\frac{\pi}{2}} \frac{\cos x}{4 - \sin^2 x}\,dx = \int_0^1 \frac{dt}{4 - t^2} = \frac{1}{4}\int_0^1 \left(\frac{1}{2-t} + \frac{1}{2+t}\right) dt$$

$\left| \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array} \right|$

$$= \frac{1}{4}\left[-\ln|2-t| + \ln|2+t|\right]_0^1$$

$$= \frac{1}{4}\left(\ln 3 - (-\ln 2 + \ln 2)\right)$$

$$= \underline{\underline{\frac{\ln 3}{4}}}$$

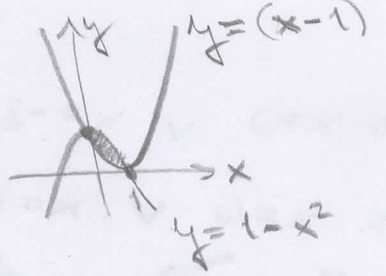
$$\frac{1}{4-t^2} = \frac{1}{(2-t)(2+t)} = \frac{A}{2-t} + \frac{B}{2+t}$$

$$1 = A(2+t) + B(2-t)$$

$\underline{t=2}$: $1 = 4A \Rightarrow A = \frac{1}{4}$

$\underline{t=-2}$: $1 = 4B \Rightarrow B = \frac{1}{4}$

(6.)



$$\int_0^1 (1-x^2)\,dx - \int_0^1 (x^2 - 2x + 1)\,dx$$

$$= \int_0^1 (-2x^2 + 2x)\,dx = \left[-2\frac{x^3}{3} + x^2\right]_0^1$$

$$= -\frac{2}{3} + 1 = \underline{\underline{\frac{1}{3}}}$$