1.

$$\begin{pmatrix}1&2&3\\2&3&4\\1&4&a\end{pmatrix} \sim \begin{pmatrix}1&2&3\\0&-1&-2\\0&2&a-3\end{pmatrix} \sim \begin{pmatrix}1&2&3\\0&-1&-2\\0&0&a-7\end{pmatrix}$$

$a=7 \Rightarrow$ vektory jsou LZ

$$\begin{pmatrix}1\\4\\7\end{pmatrix} \overset{?}{=} a_1\cdot\begin{pmatrix}1\\2\\3\end{pmatrix} + a_2\cdot\begin{pmatrix}2\\3\\4\end{pmatrix}$$

$$\left(\begin{array}{cc|c}1&2&1\\2&3&4\\3&4&7\end{array}\right) \sim \left(\begin{array}{cc|c}1&2&1\\0&-1&2\\0&-2&4\end{array}\right)$$

$a_2=-2$

$a_1-4=1$

$a_1=5$

$$\boxed{\vec{u}_3 = 5\vec{u}_1 - 2\vec{u}_2}$$

2.

ROZVOJ PODLE 4. SLOUPCE

$$\begin{vmatrix}2&4&9&0&5\\4&4&5&2&6\\0&7&3&0&5\\0&1&0&0&0\\0&2&4&0&6\end{vmatrix} = 2\cdot(-1)^{4+2}\cdot\begin{vmatrix}2&4&9&5\\0&7&3&5\\0&1&0&0\\0&2&4&6\end{vmatrix} =$$

ROZVOJ PODLE 3. ŘÁDKU

ROZVOJ PODLE 1. SLOUPCE

$$= 2\cdot1\cdot(-1)^{3+2}\cdot\begin{vmatrix}2&9&5\\0&3&5\\0&4&6\end{vmatrix} = (-2)\cdot2\cdot(-1)^{1+1}\cdot\begin{vmatrix}3&5\\4&6\end{vmatrix} =$$

$$= (-4)\cdot(3\cdot6-5\cdot4) = (-4)\cdot(-2) = \underline{\underline{8}}$$

3.

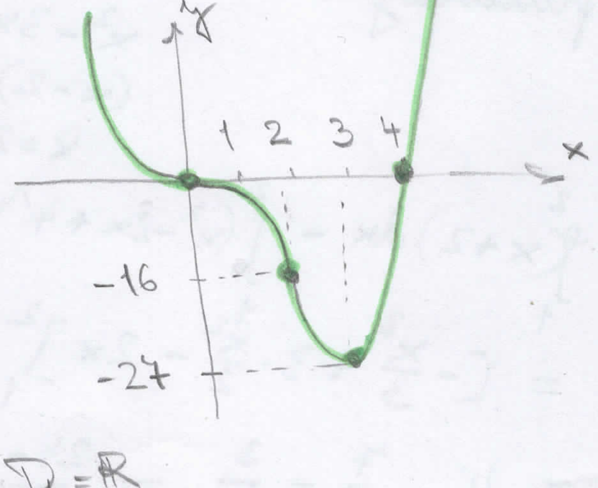
$f(x) = x^4-4x^3 = x^3(x-4)$

x	0	4	3	2
$f(x)$	0	0	-27	-16

$f'(x) = 4x^3-12x^2 = 4x^2(x-3)$

$\searrow \quad \searrow \quad \nearrow$

$f'<0 \quad 0 \quad f'<0 \quad 3 \quad f'>0$



$f(3) = 3^3\cdot(3-4) = -27$

lok. minimum: $[3;-27]$

$f''(x) = 12x^2-24x = 12x(x-2)$

$\cup \quad \cap \quad \cup$

$f''>0 \quad 0 \quad f''<0 \quad 2 \quad f''>0$

$D_f = \mathbb{R}$

$H_f = \langle -27, +\infty)$

$f(2) = 2^3(2-4) = -16$

(4.) $A = [2; -3]$
$B = [8; 0]$

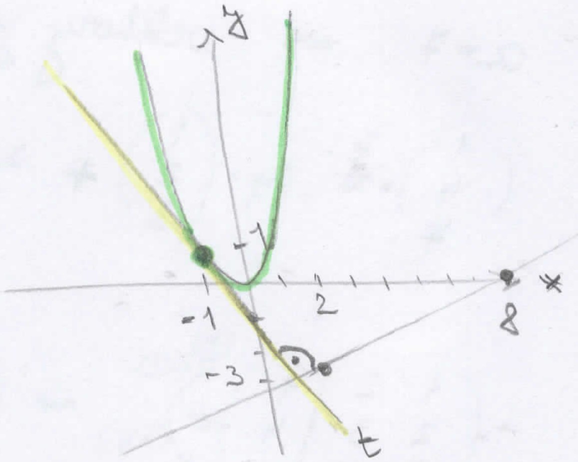
$f(x) = x^2$
$f'(x) = 2x$

$\vec{AB} = B - A = (6, 3) = \vec{n}_t$

$t: 6x + 3y + c = 0$

$y = -2x - \frac{c}{3}$

směrnice tečny $k = -2 \stackrel{!}{=} f'(x_0)$
$-2 = 2x$
$x_0 = -1$



tečný bod $[-1; 1] = T$

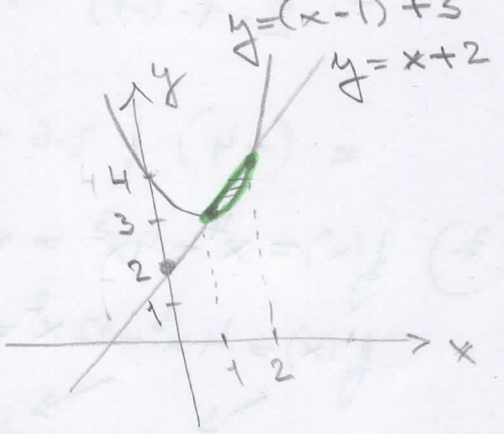
$T \in t \Rightarrow 1 = 2 + k \Rightarrow k = -1$

tečna: $\boxed{y = -2x - 1}$

(5.) $\int_{-\frac{1}{2}}^{0} (2x+1)^{49}\, dx = \frac{1}{2}\int_0^1 t^{49}\, dt = \frac{1}{2} \cdot \left[\frac{t^{50}}{50}\right]_0^1 = \underline{\underline{\frac{1}{100}}}$

$\left| \begin{array}{l} 2x+1 = t \\ 2\,dx = dt \\ dx = \frac{1}{2} dt \end{array} \right|$

(6.) $y = x^2 - 2x + 4 = (x-1)^2 + 3$
$y = x + 2$



průsečíky křivek: $x^2 - 2x + 4 = x + 2$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$x = 2 \lor x = 1$

$\int_1^2 (x+2)\, dx - \int_1^2 (x^2 - 2x + 4)\, dx = \int_1^2 (-x^2 + 3x - 2)\, dx =$

$= \left[-\frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - 2x\right]_1^2 = -\frac{8}{3} + 6 - 4 - \left(-\frac{1}{3} + \frac{3}{2} - 2\right) =$

$= 4 - \frac{7}{3} - \frac{3}{2} = \frac{24 - 14 - 9}{6} = \underline{\underline{\frac{1}{6}}}$