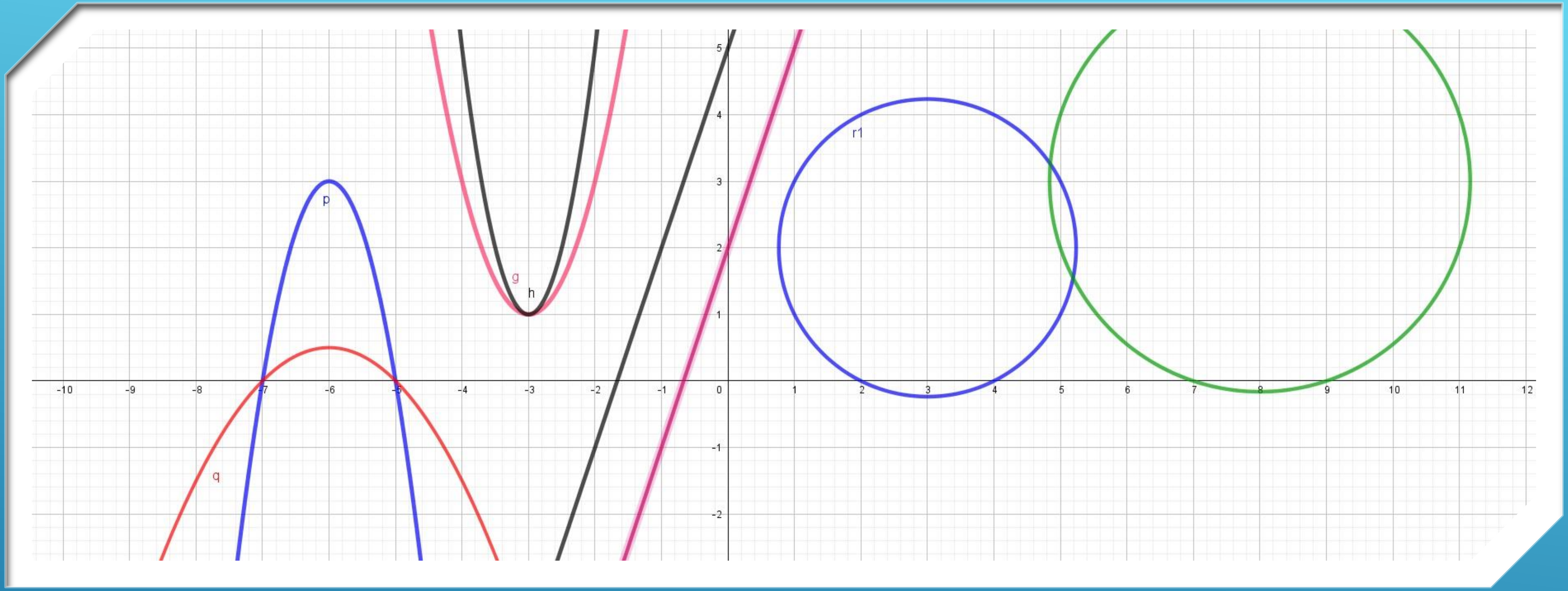
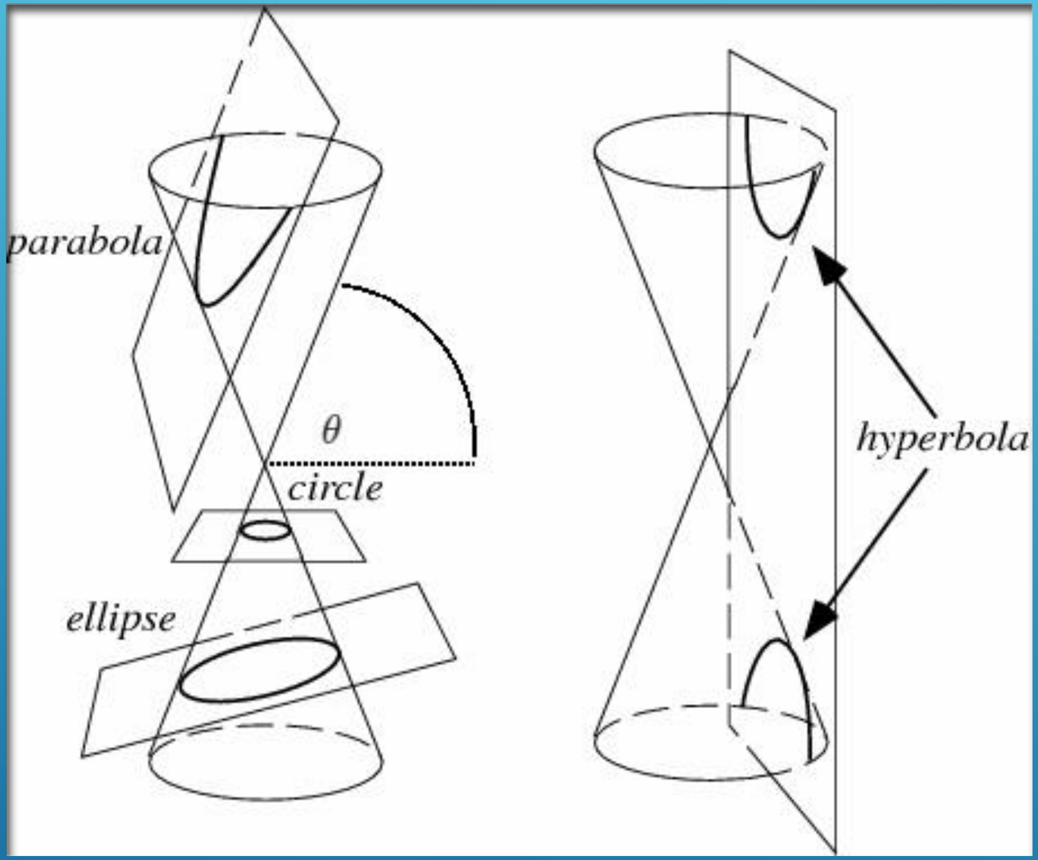


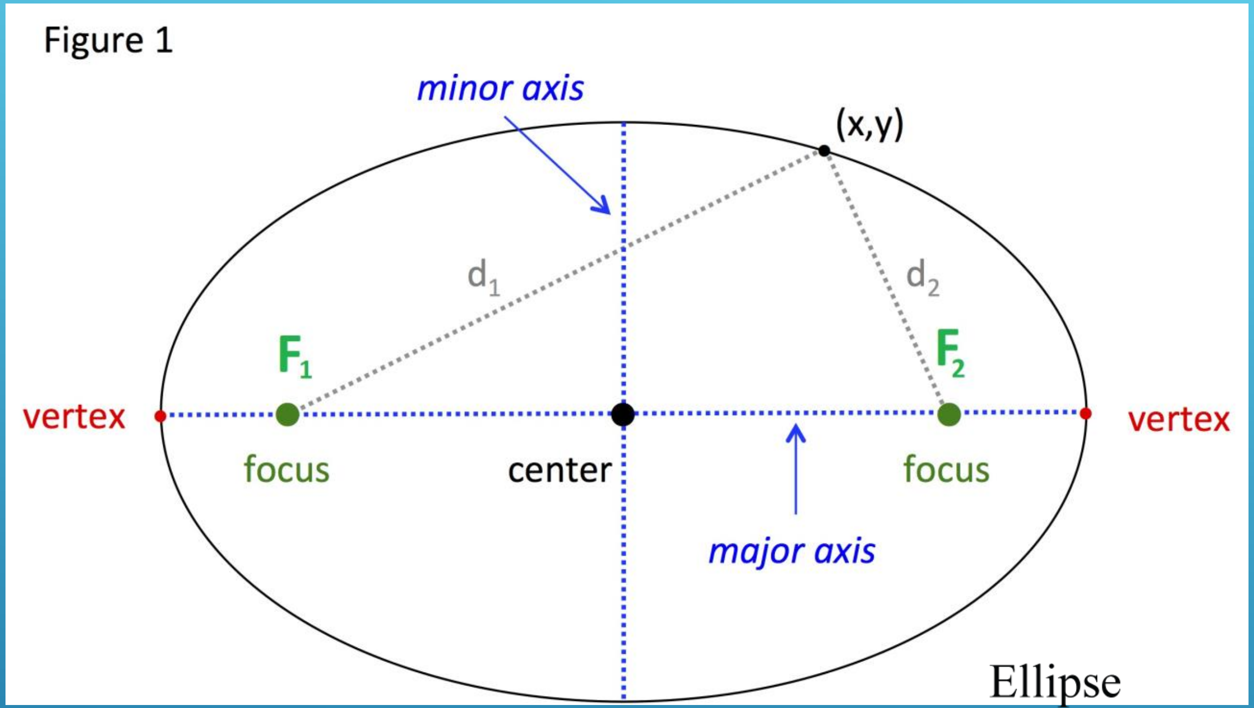
CONIC SECTIONS (CONICS)

Filip Konopka



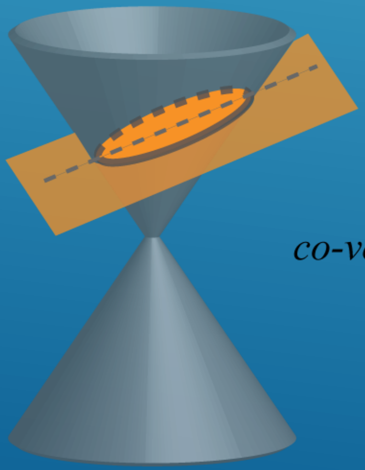
- The conic sections are **curves** obtained by the **intersection** of **a right circular cone** and **a plane**. According to the angle of intersection the conic is an ellipse, a parabola or a hyperbola. A circle is also a conic – it's a special case of an ellipse.

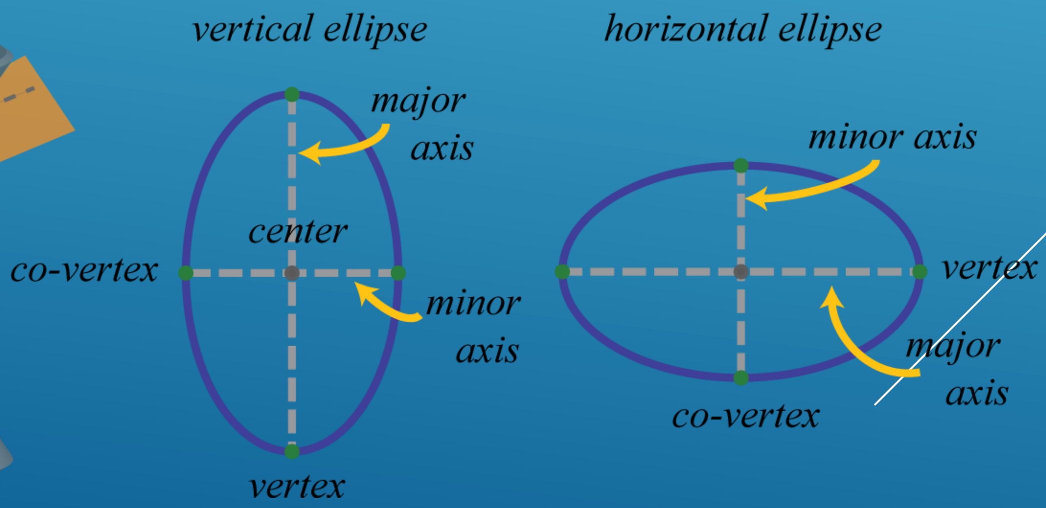
CONIC SECTIONS



- Ellipse is a closed curve which is symmetrical about both its axes.
- Fixed points F1 and F2 are called **foci** of an ellipse.
- The line segment through the foci is the **major axis**. Perpendicular to the major axis through the centr is the **minor axis**.
- The points where the axes cut the ellipse are the **vertices**.
- The midpoint of the vertices is the **centre** of the ellipse.

ELLIPSE





- Given two fixed points F1, F2 called the foci and a distance 2a which is greater than the distance between the foci. The ellipse is the **set of points** P such that the sum of the distances |PF1| and |PF2| is equal to 2a.

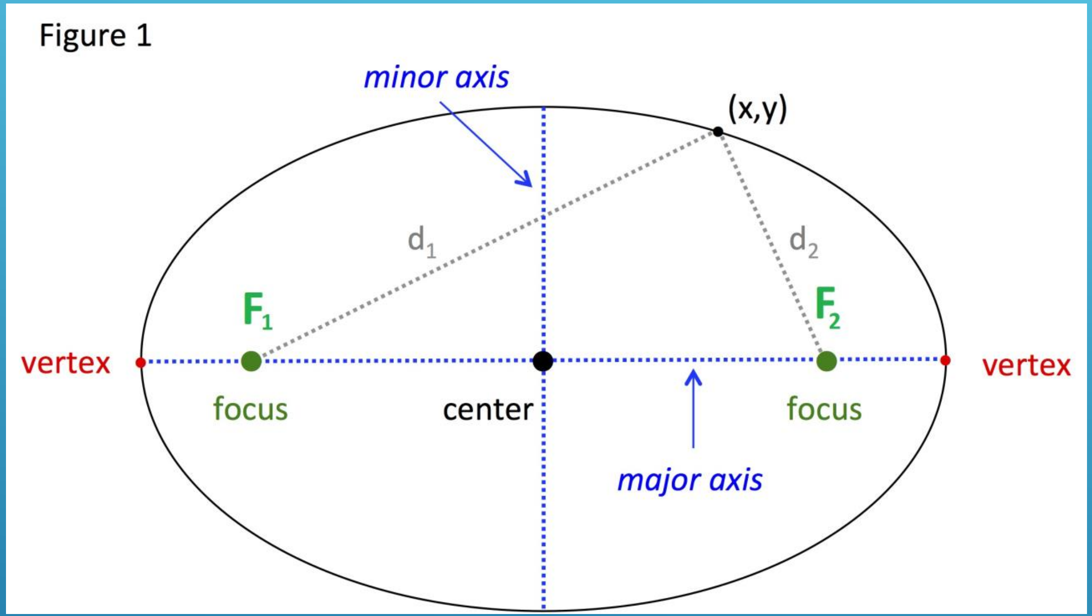


Figure 1

ELLIPSE

$$E = \{P \in \mathbb{R}^2 \ | \ |PF_2| + |PF_1| = 2a\}$$

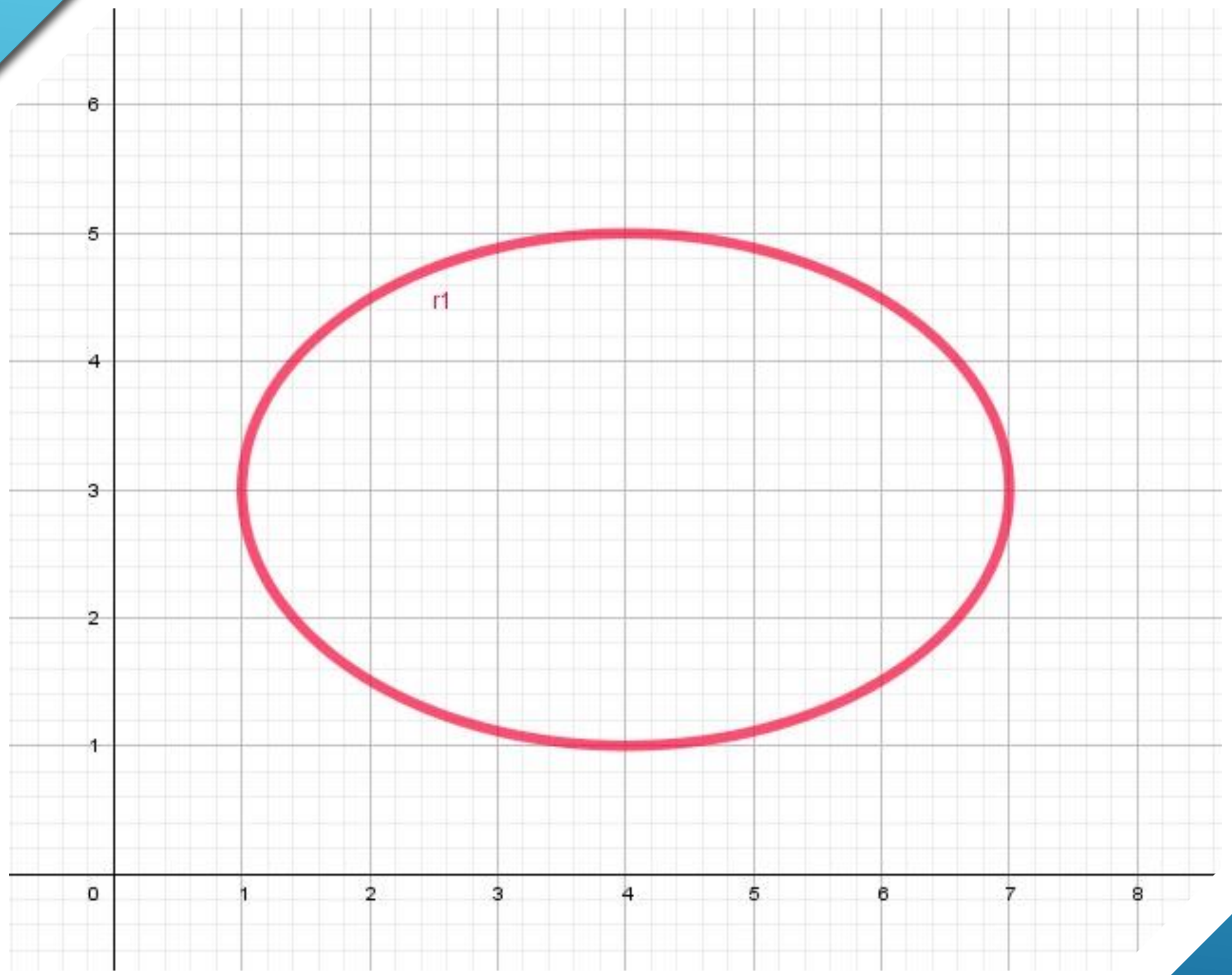
General equation of ellipse $Ax^2 + By^2 + Cx + Dy + E = 0$ $A,B > 0$

Standard equation of ellipse $$\frac{(x - x_\circ)^2}{a^2} + \frac{(y - y_\circ)^2}{b^2} = 1$$

Point [x_0, y_0] is the centre of ellipse and a,b are length of axis.

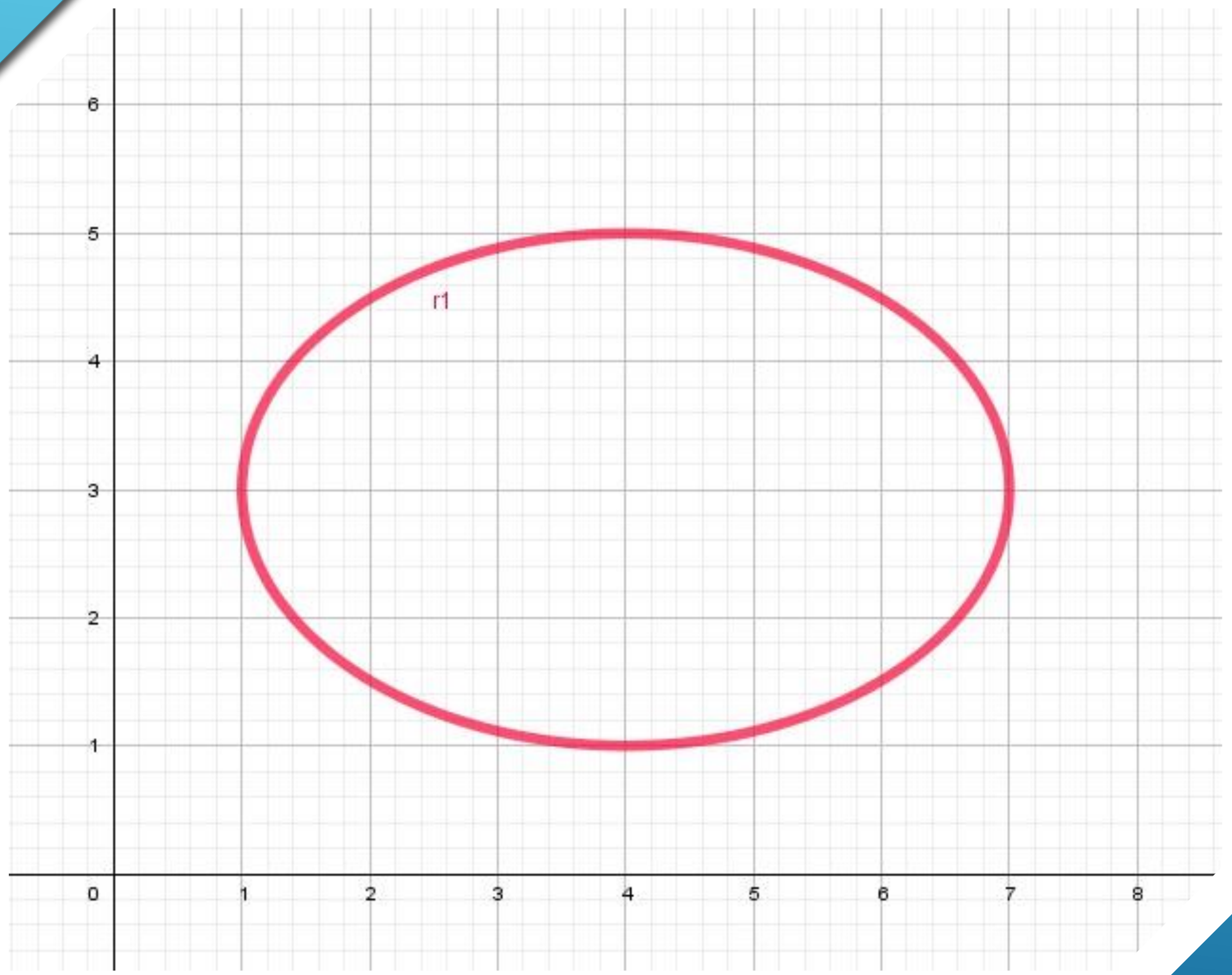
EQUATIONS OF ELLIPSE

- $x^2 + 36y^2 - 1 = 0$ is equation of a real ellipse.
- $x^2 + 36y^2 + 1 = 0$ is not equation of a real ellipse.



TASKS

- Find the centre of the ellipse.
- Find the lengths of axis of this ellipse.
- Find the equation of this ellipse.



TASKS

- Find the centre of the ellipse.
- Find the lengths of axis of this ellipse.
- Find the equation of this ellipse.

$$\frac{(x-4)^2}{9}+\frac{(y-3)^2}{4}=1$$

- Find the centre and vertices of the ellipse. Sketch the ellipse.

$$4x^2 + 25y^2 = 100$$

TASKS FOR STUDENTS

- Find the centre and vertices of the ellipse. Sketch the ellipse.

$$4x^2 + 25y^2 = 100$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

TASKS FOR STUDENTS

- Find the centre and vertices of the ellipse. Sketch the ellipse.

$$4x^2 + 25y^2 = 100$$

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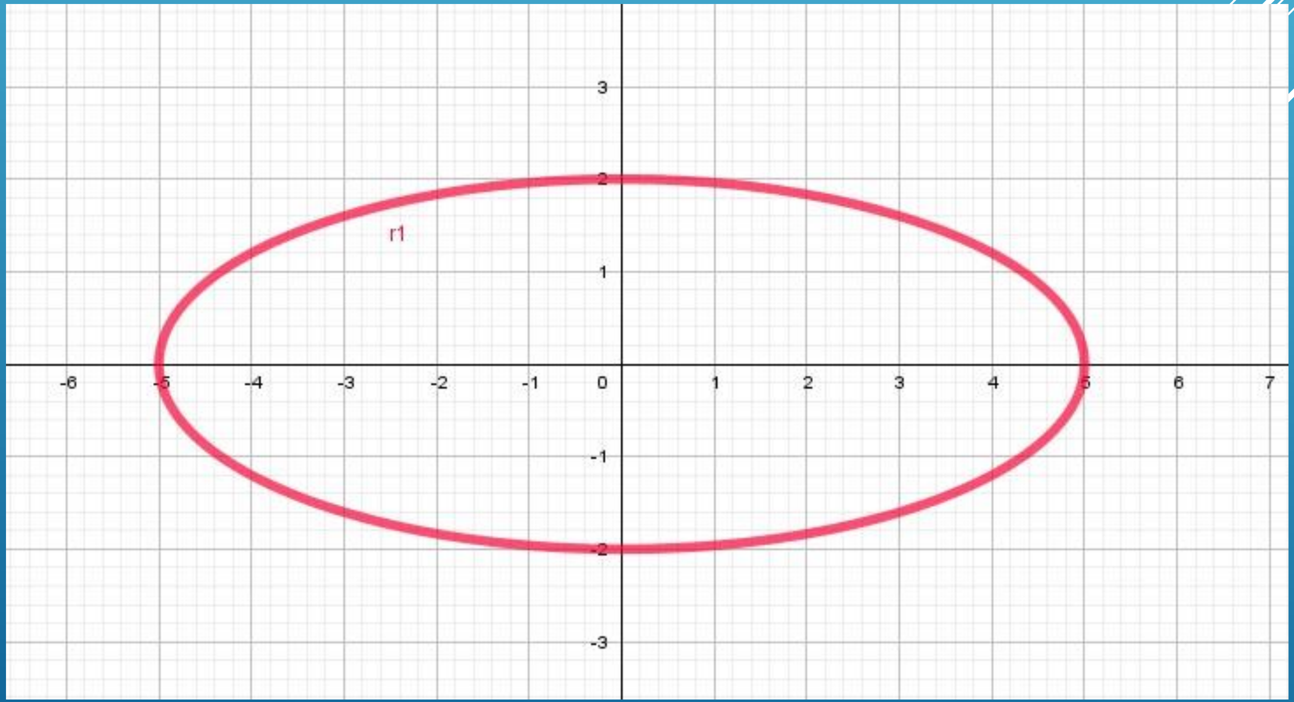
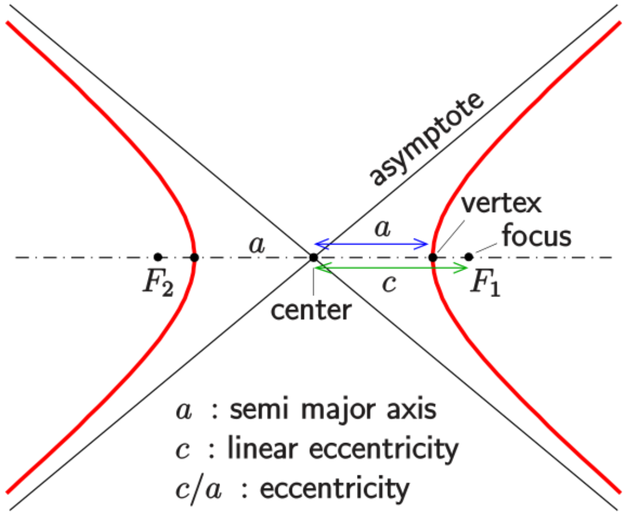
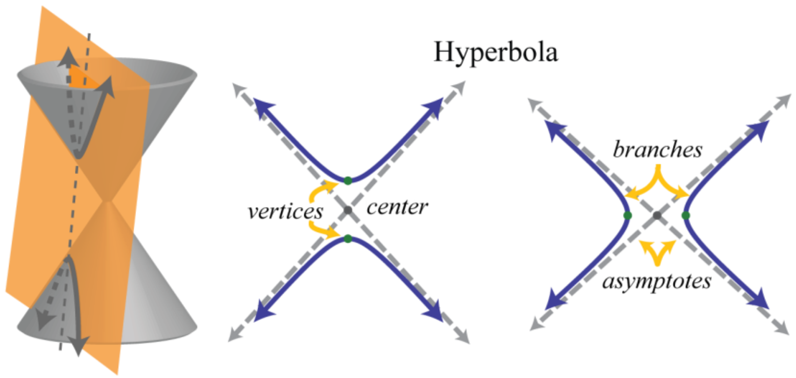


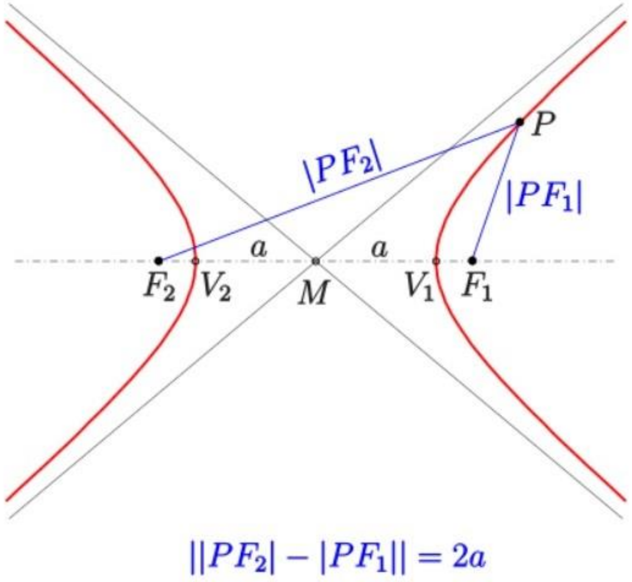
TASKS FOR STUDENTS





- Hyperbola is a **two-branched** open **curve**
- Fixed points F1 and F2 are called **foci** of a hyperbola
- The line through the F1 and F2 is the **transverse axis** and the line through the centre perpendicular to the transverse axis is the **conjugate axis**.
- The points the transverse axis cuts the hyperbola and the vertices
- The midpoint of the vertices is the **centre of the hyperbola**
- The two separate parts of the hyperbola are the two **branches**.
- Every hyperbola has two **asymptotes** which cross the centre of hyperbola. Hyperbola approaches the asymptotes.

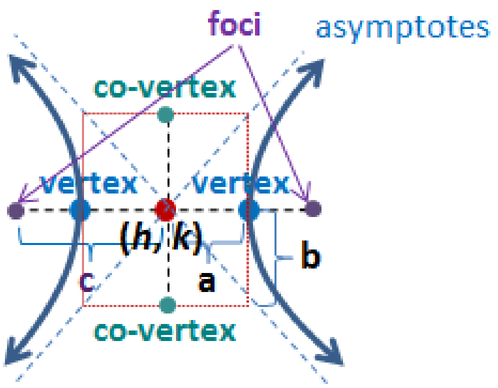
HYPERBOLA

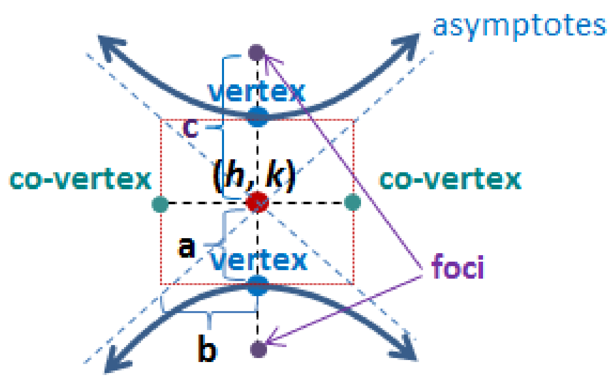


A **hyperbola** is a set of points, such that for any point P of the set, the absolute difference of the distances $|PF_1|$, $|PF_2|$ to two fixed points F_1, F_2 (the *foci*) is constant, usually denoted by $2a,\ a > 0$:

$$H = \{P : ||PF_2| - |PF_1|| = 2a\} .$$

DEFINITION OF HYPERBOLA

<table>
<tr><th>Horizontal Hyperbola
(x^2 comes first)</th><th>Vertical Hyperbola
(y^2 comes first)</th></tr>
<tr><td>
At (0, 0): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
General: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$a^2 + b^2 = c^2$
Center: (h, k) **Foci:** $(h \pm c, k)$
Vertices: $(h \pm a, k)$ **Co-Vertices:** $(h, k \pm b)$
Asymptotes: $y - k = \pm\frac{b}{a}(x-h)$


</td><td>
At (0, 0): $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
General: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
$a^2 + b^2 = c^2$
Center: (h, k) **Foci:** $(h, k \pm c)$
Vertices: $(h, k \pm a)$ **Co-Vertices:** $(h \pm b, k)$
Asymptotes: $y - k = \pm\frac{a}{b}(x-h)$


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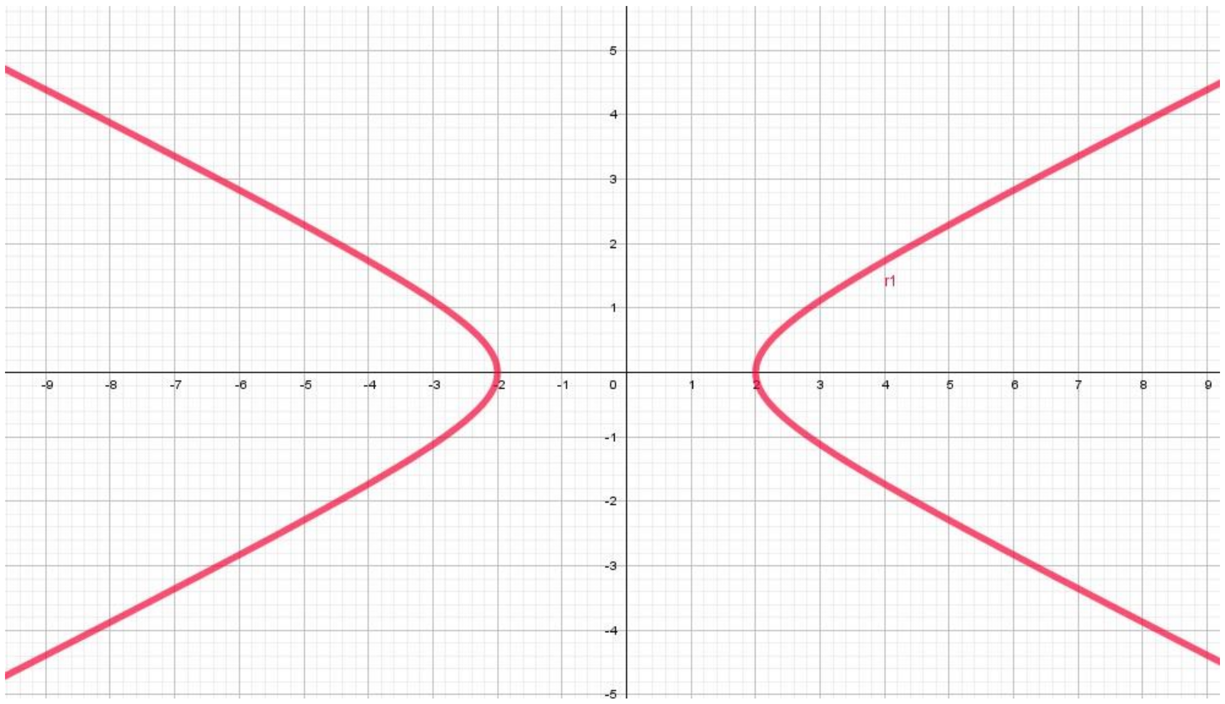
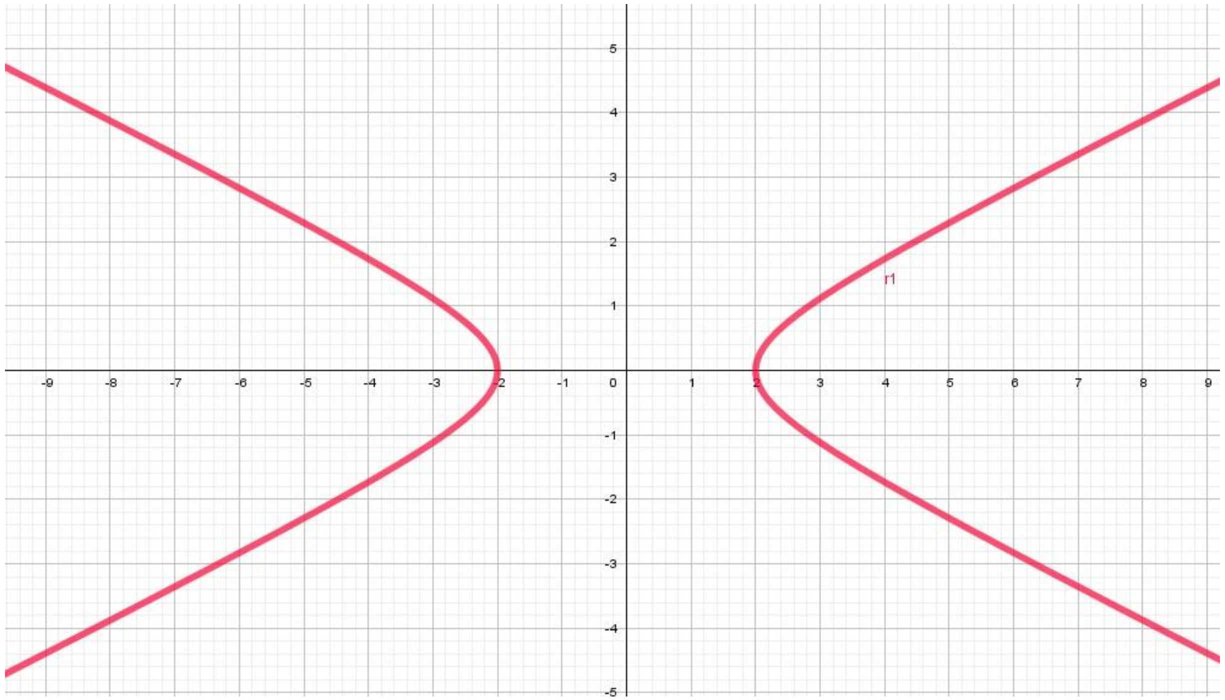


TASK FOR STUDENTS

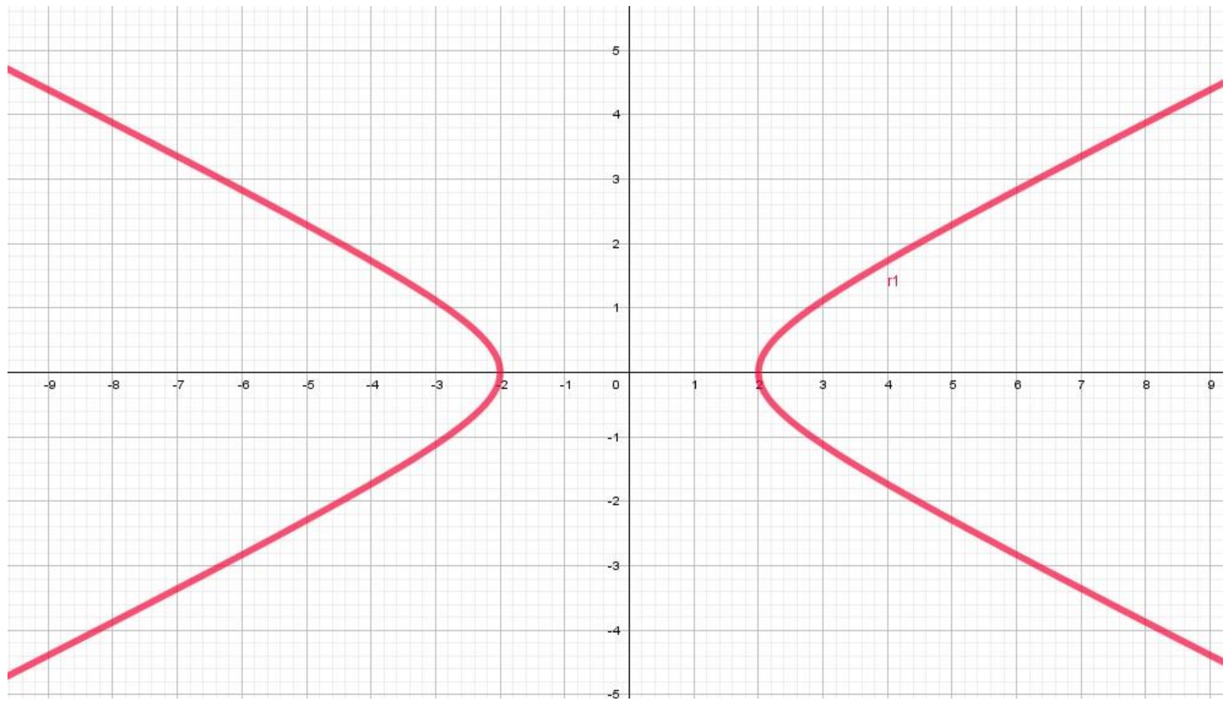
- Determine equation of a hyperbola given its graph.
- Determine equations of asymptotes of this hyperbola.



TASK FOR STUDENTS

- Determine equation of a hyperbola given its graph.
- Determine equations of asymptotes of this hyperbola.

$$\frac{x^2}{4} - y^2 = 1$$



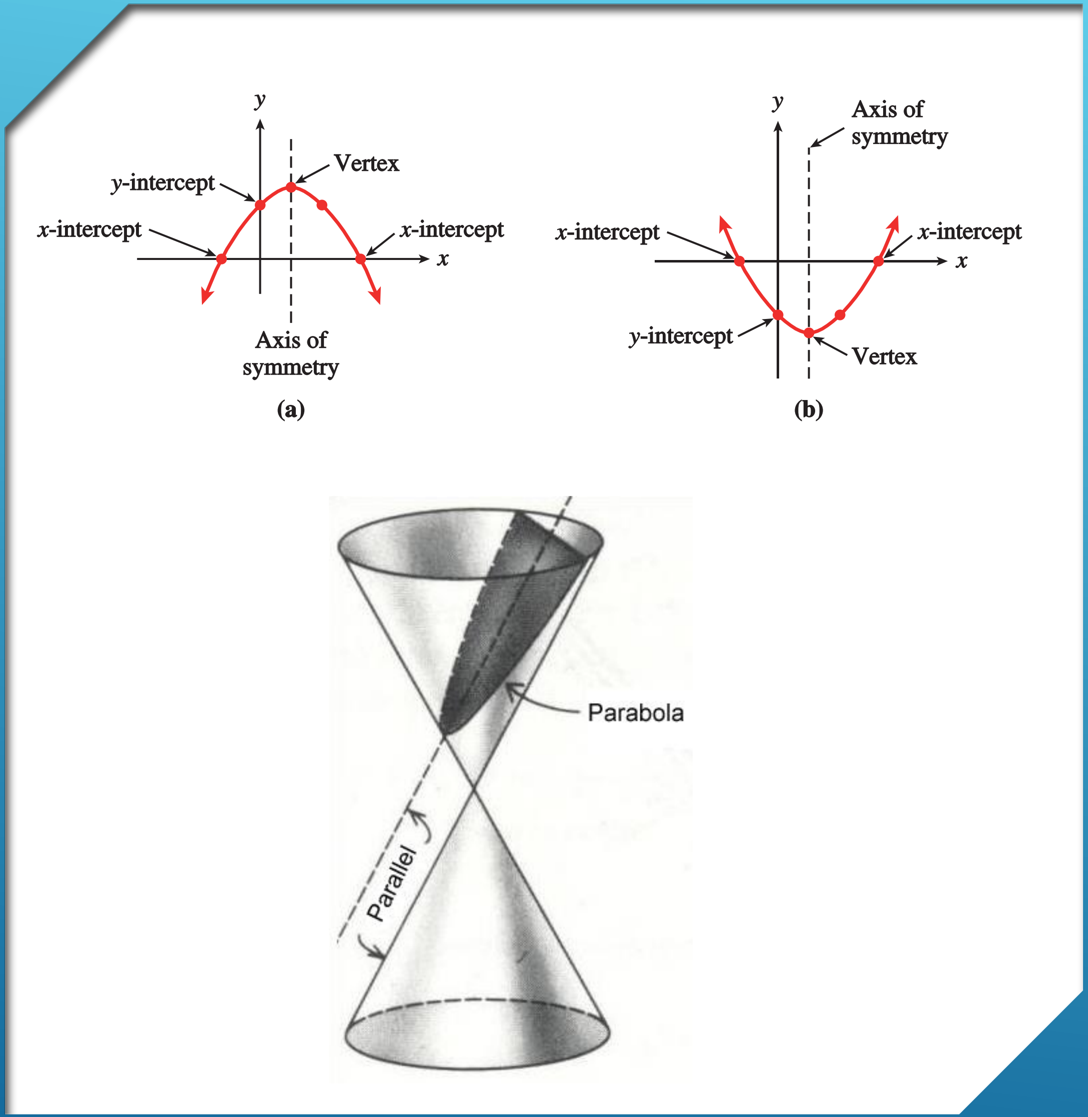
TASK FOR STUDENTS

- Determine equation of a hyperbola given its graph.
- Determine equations of asymptotes of this hyperbola.

$$\frac{x^2}{4} - y^2 = 1$$

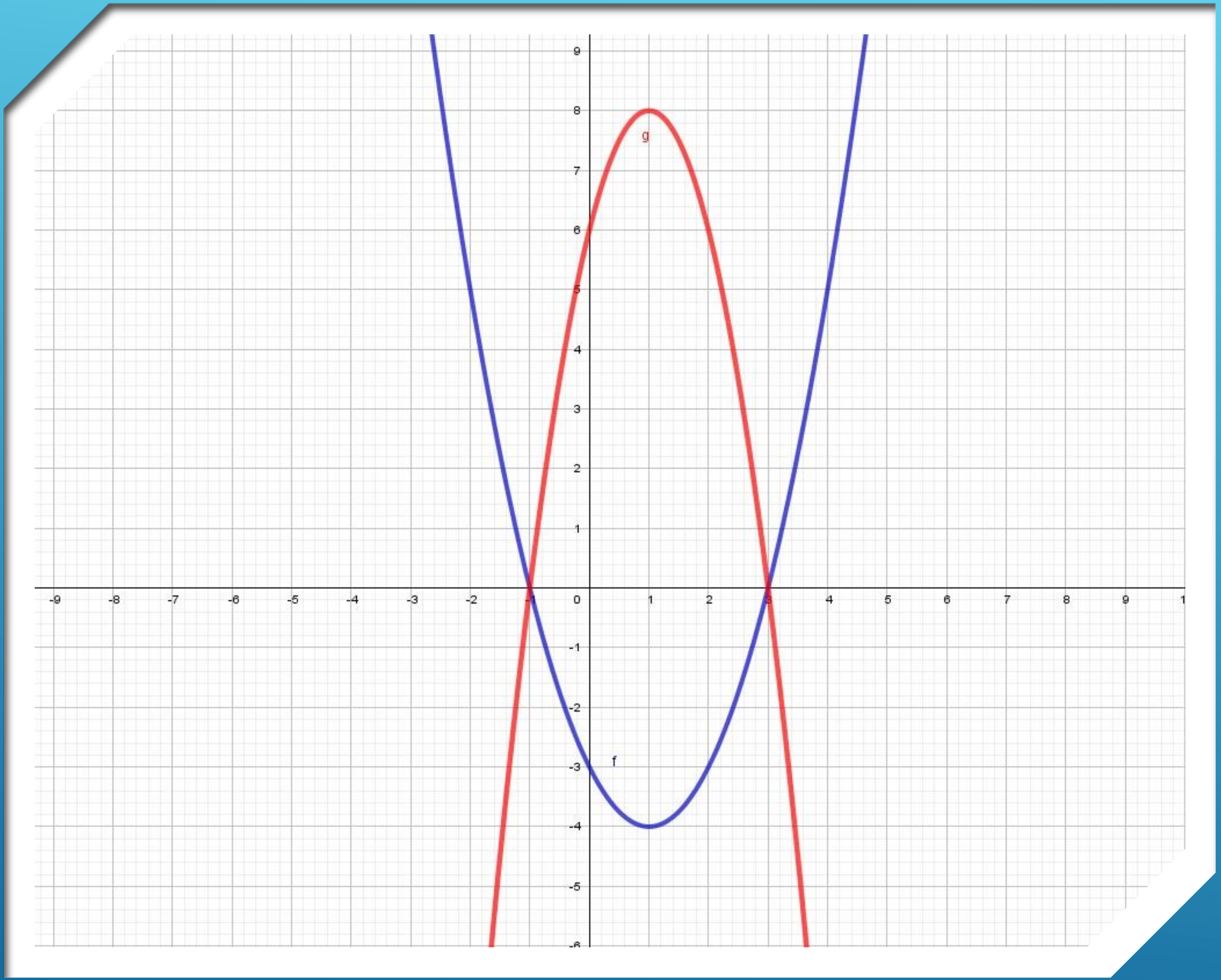
$y = \frac{1}{2}x$

$y = -\frac{1}{2}x$



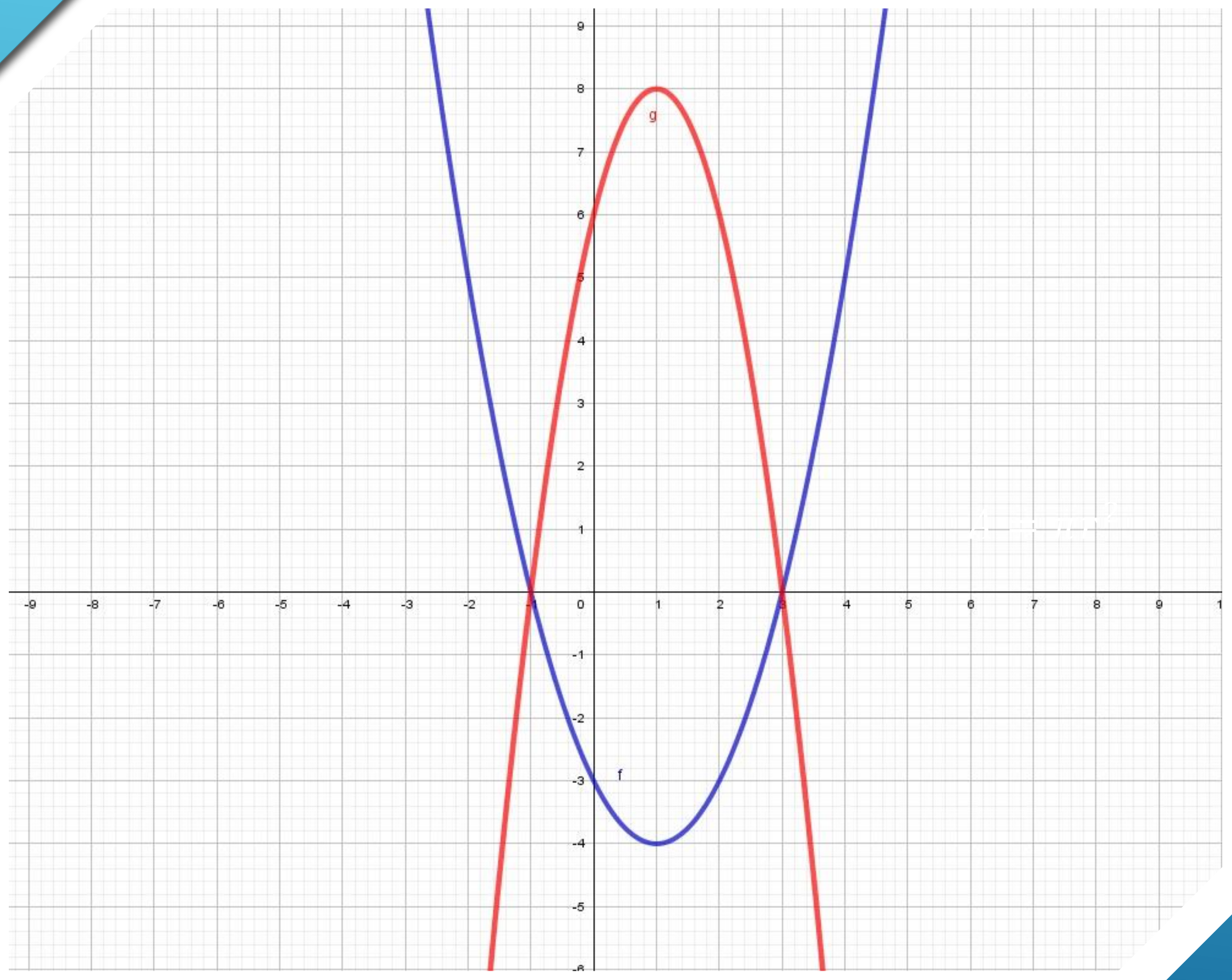
- Parabola is an open curve. It is the **locus of a point** that moves in a plane so as to be **equidistant from a fixed line and a fixed point**.
- The fixed line is called the **directrix**.
- The fixed point is called the **focus**.
- The line through the focus perpendicular to the directrix is the **axis** of the parabola.
- The point where the axis cuts the parabola is the **vertex**. It is possible to take the vertex as the origin.

PARABOLA



TASK FOR STUDENTS

- Find equations of these parabolas.



TASK FOR STUDENTS

- Find equations of these parabolas.

$$y = (x+1)(x-3)$$
$$y = -2(x+1)(x-3)$$

- Find the vertex of parabola $y = x^2 - 2x + 5$

TASK

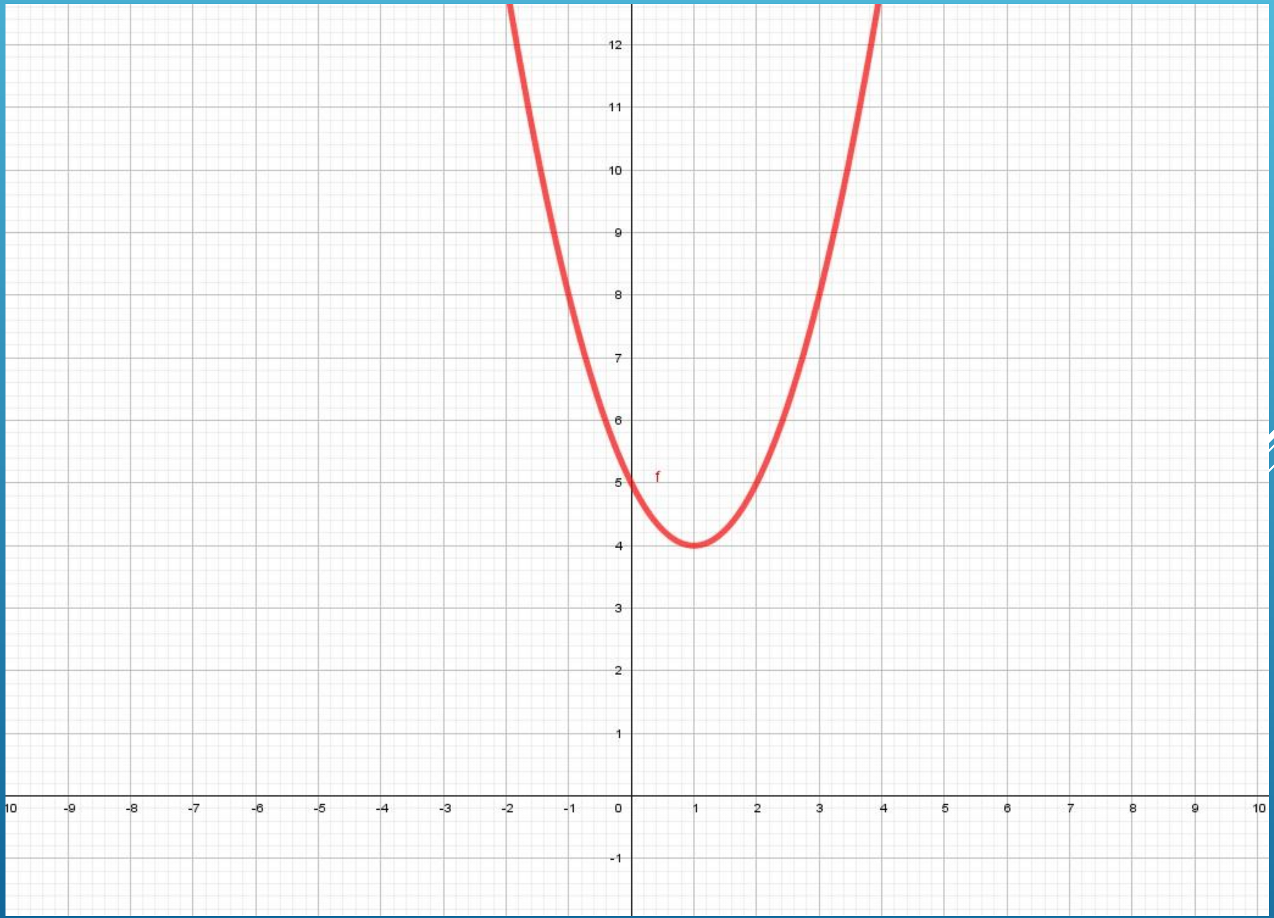
- Find the vertex of parabola $y = x^2 - 2x + 5$
- $y = x^2 - 2x + 1 - 1 + 5$

TASK

- Find the vertex of parabola $y = x^2 - 2x + 5$
- $y = x^2 - 2x + 1 - 1 + 5$
- $y = (x - 1)^2 + 4$

TASK

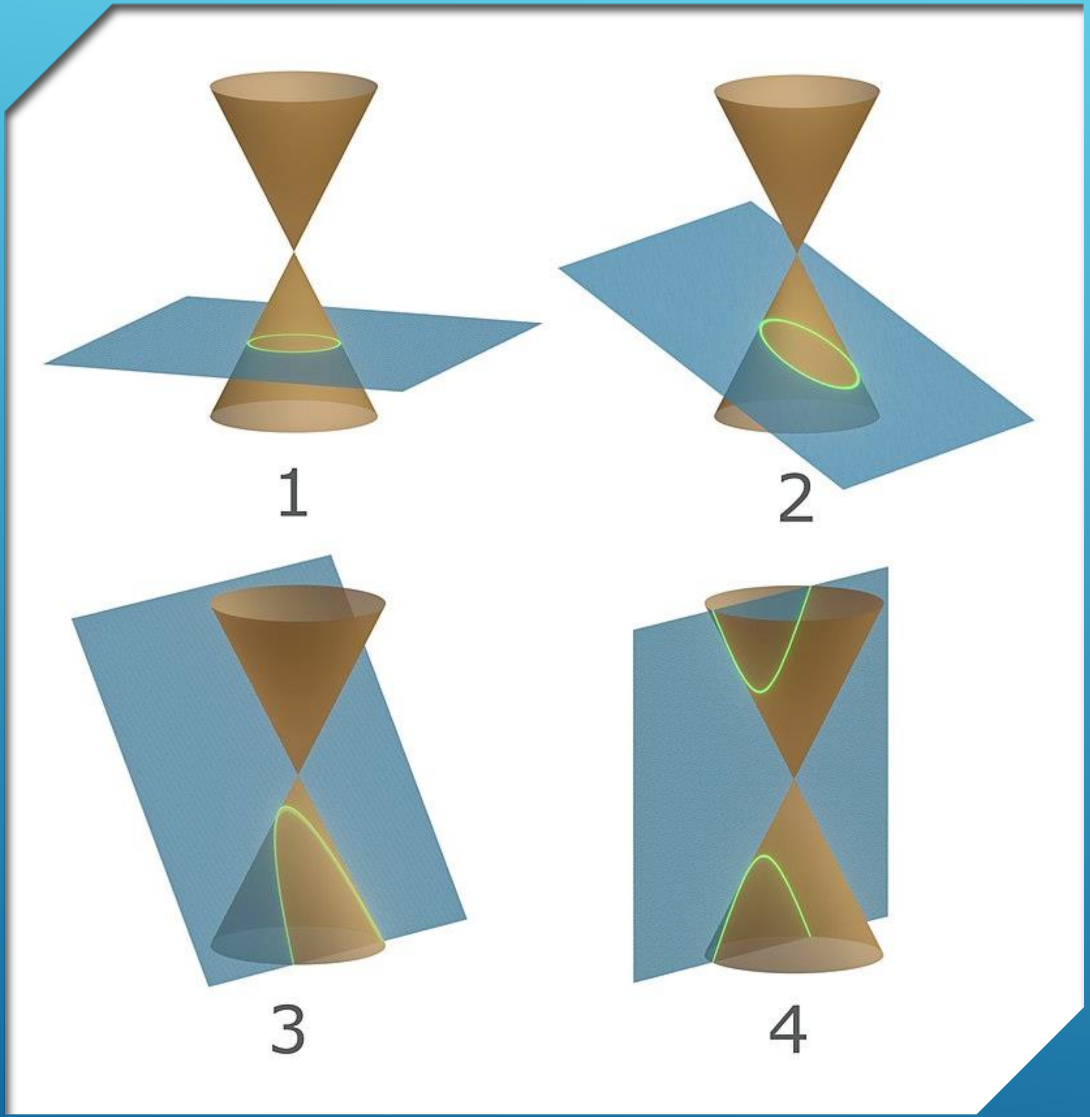
- Find the vertex of parabola $y = x^2 - 2x + 5$
- $y = x^2 - 2x + 1 - 1 + 5$
- $y = (x-1)^2 + 4$
- The vertex is [1;4]



TASK

- Find an equation for the circle with radius 2 and centre at [3; 4].
- Find an equation for the parabola which passes through the point [1; 3]. and has vertex at [2; 4].
- Find an equation for the hyperbola with centre at [0; 0] such that major axis is paraller to x-axis and the length of major axis is 2 and the length of minor axis is 1.
- Find an equation for the ellipse with centre at [-3; 5] such that major axis is paraller to y-axis and the length of major axis is 3 and the length of minor axis is 4.

TASK FOR STUDENTS



THANK YOU FOR ATTENTION