(1a) $F(x,y) = \left(\dfrac{2x(1-e^y)}{(1+x^2)^2} , \dfrac{e^y}{1+x^2} \right)$

$$\frac{\partial F_2}{\partial x}(x,y) = e^y \cdot (-1) \cdot (1+x^2)^{-2} \cdot 2x = -\frac{2x\, e^y}{(1+x^2)^2}$$

$$\frac{\partial F_1}{\partial y}(x,y) = \frac{\partial}{\partial y}\left(\frac{2x}{(1+x^2)^2} - \frac{2x}{(1+x^2)^2} \cdot e^y \right)$$

$$= 0 - \frac{2x}{(1+x^2)^2} e^y$$

$$\Longrightarrow \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \Longrightarrow F \text{ je potenciální}$$

(1b) $F(x,y,z) = \left(y^2 + 2xz^2 - 1 , 2xy , 2x^2 z + z^3 \right)$

$\dfrac{\partial F_1}{\partial z} = 4xz$ $\qquad \dfrac{\partial F_1}{\partial y} = 2y$ $\qquad \dfrac{\partial F_2}{\partial z} = 0$

$\dfrac{\partial F_3}{\partial x} = 4xz$ $\qquad \dfrac{\partial F_2}{\partial x} = 2y$ $\qquad \dfrac{\partial F_3}{\partial y} = 0$

$$\frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x} \wedge \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \wedge \frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$$

$\Longrightarrow$ F je potenciální

(2.) $F(x,y) = \left(x e^{y} + 1,\ y + \dfrac{x^2 e^{y}}{2}\right)$

Vektorové pole F je potenciální, neboť $\dfrac{\partial F_2}{\partial x} = \dfrac{\partial F_1}{\partial y}$

$\Rightarrow$ křivkový integrál $\int\limits_{\mathcal{K}} \vec{F}\cdot d\vec{r}$ nezávisí

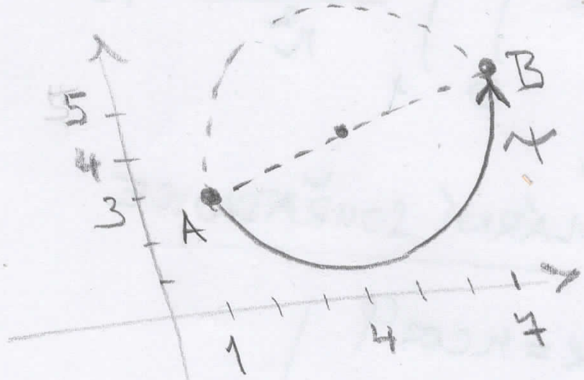
na integrační cestě

tedy $\int\limits_{K_1} \vec{F}\cdot d\vec{r} = \int\limits_{K_2} \vec{F}\cdot d\vec{r}$ $\quad \forall K_1, K_2 : [a,b] \to \mathbb{R}^2$

křivky takové, že

$K_1(a) = K_2(a)$

a $K_1(b) = K_2(b)$



úsečka z bodu $[1;3]$ do $[7;5]$: $\quad \begin{vmatrix} x = 1 + 6t \\ y = 3 + 2t \\ t \in [0,1] \end{vmatrix}$

směrový vektor $\vec{AB} = B - A = (6,2)$

$$\int\limits_{\mathcal{K}} \vec{F}\cdot d\vec{r} = \int_0^1 \left(\left((1+6t)e^{3+2t} + 1\right)\cdot 6 + (3+2t) + \frac{(1+6t)^2 e^{3+2t}}{2}\cdot 2\right) dt$$

$$= \int_0^1 \left((6+36t)e^{3+2t} + 6 + 3 + 2t + (1+6t)^2 e^{3+2t}\right) dt$$

$$= \ldots \text{ PER-PARTES } = 14 - \frac{1}{2}e^{3} + \frac{49}{2}e^{5}$$

(2) Jiný možný postup (jednodušší)

Najdeme potenciál, tj. funkci U takovou, že $\nabla U = F$

a poté $\boxed{\int_k \vec{F} \cdot d\vec{r} = U(B) - U(A)}$

$$\frac{\partial U}{\partial x} = F_1 \implies \int (x e^y + 1)\, dx = \frac{x^2}{2} \cdot e^y + x + c_1(y)$$

$$\frac{\partial U}{\partial y} = F_2 \implies \int \left(y + \frac{x^2 e^y}{2}\right) dy = \frac{y^2}{2} + \frac{x^2}{2} e^y + c_2(x)$$

Potenciál pole $\vec{F}$ je $U(x,y) = \frac{x^2}{2} e^y + x + \frac{y^2}{2} + c$, $c \in \mathbb{R}$

$$\int_k \vec{F} \cdot d\vec{r} = U(7,5) - U(1,3) = \frac{49}{2} e^5 + 7 + \frac{25}{2} + c$$

$$- \left(\frac{1}{2} e^3 + 1 + \frac{9}{2} + c \right) =$$

$$= \underline{\underline{\frac{49}{2} e^5 - \frac{1}{2} e^3 + 14}}$$