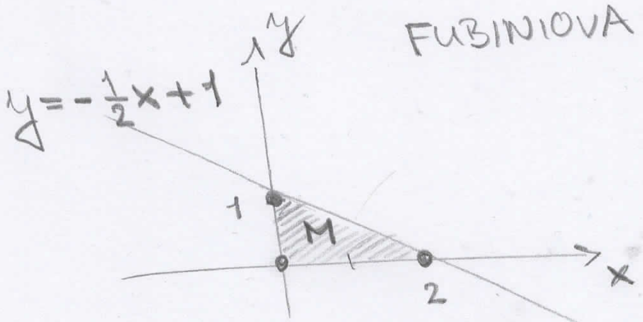
1.

$$\iint_M (5x^2-2xy)\,dx\,dy = \int_0^2\int_0^{-\frac{1}{2}x+1}(5x^2-2xy)\,dy\,dx =$$

FUBINIOVA VĚTA



$$= \int_0^2 \left[5x^2y - xy^2\right]_0^{-\frac{1}{2}x+1} dx =$$

$$= \int_0^2 \left(5x^2\left(-\frac{x}{2}+1\right) - x\left(-\frac{x}{2}+1\right)^2\right) dx$$

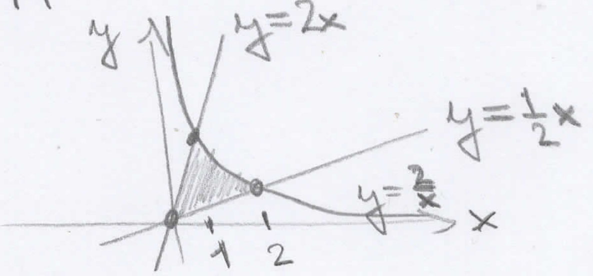
$$= \int_0^2 \left(-\frac{5}{2}x^3 + 5x^2 - x + x^2 - \frac{x^3}{4}\right) dx$$

$$= \int_0^2 \left(-\frac{11}{4}x^3 + 6x^2 - x\right) dx = \left[-\frac{11}{16}x^4 + 2x^3 - \frac{x^2}{2}\right]_0^2$$

$$= -11 + 16 - 2 = \underline{\underline{3}}$$

2.

$$\iint_M (x^2+y)\,dx\,dy \overset{F.V.}{=} \int_0^1\int_{\frac{x}{2}}^{2x}(x^2+y)\,dy\,dx + \int_1^2\int_{\frac{x}{2}}^{\frac{2}{x}}(x^2+y)\,dy\,dx$$



$$= \int_0^1 \left[x^2y + \frac{y^2}{2}\right]_{\frac{x}{2}}^{2x} dx +$$

$$+ \int_1^2 \left[x^2y + \frac{y^2}{2}\right]_{\frac{x}{2}}^{\frac{2}{x}} dx$$

$$= \int_0^1 \left(2x^3 + 2x^2 - \frac{x^3}{2} - \frac{x^2}{8}\right) dx + \int_1^2 \left(2x + \frac{2}{x^2} - \frac{x^3}{2} - \frac{x^2}{8}\right) dx$$

$$= \left[\frac{x^4}{2} + 2\frac{x^3}{3} - \frac{x^4}{8} - \frac{x^3}{24}\right]_0^1 + \left[x^2 - \frac{2}{x} - \frac{x^4}{8} - \frac{x^3}{24}\right]_1^2$$

$$= \frac{1}{2} + \frac{2}{3} - \frac{1}{8} - \frac{1}{24} + 4 - 1 - 2 - \frac{1}{3} - \left(1 - 2 - \frac{1}{8} - \frac{1}{24}\right)$$

$$= 2 + \frac{1}{2} + \frac{1}{3} = \frac{12+3+2}{6} = \underline{\underline{\frac{17}{6}}}$$

2b

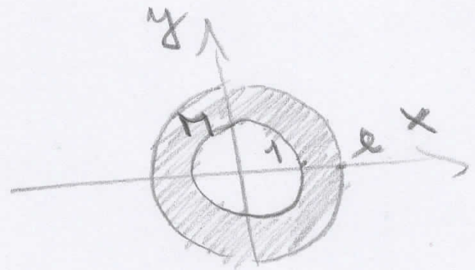
$$\iint_M (x^2+y)\,dx\,dy = \int_0^1 \int_{\frac{y}{2}}^{2y} (x^2+y)\,dx\,dy + \int_1^2 \int_{\frac{y}{2}}^{\frac{2}{y}} (x^2+y)\,dx\,dy$$

3.

$$\iint_M \frac{\ln(x^2+y^2)}{x^2+y^2}\,dx\,dy = \int_0^{2\pi} \int_1^{e} \frac{\ln r^2}{r^2} \cdot r\,dr\,d\varphi$$

POLÁRNÍ SOUŘADNICE

$$M = 1 \le x^2+y^2 \le e^2$$



$$\left| \begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \end{array} \right|$$

$J = r$ JAKOBIÁN

$$= \int_0^{2\pi} 1\,d\varphi \cdot \int_1^{e} \frac{2\ln r}{r}\,dr = 2\pi \cdot 2 \int_1^{e} \frac{\ln r}{r}\,dr$$

FUBINIOVA VĚTA

$$= \left| \begin{array}{l} \ln r = t \\ \frac{1}{r}dr = dt \end{array} \right| 4\pi \cdot \int_0^1 t\,dt = 4\pi \cdot \left[\frac{t^2}{2}\right]_0^1 = \underline{\underline{2\pi}}$$