1.

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & a & 1 \\ 1 & -1 & a \end{pmatrix}$$

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$$\det A = \begin{vmatrix} 1 & 0 & 2 \\ 2 & a & 1 \\ 1 & -1 & a \end{vmatrix} = 1\cdot(-1)^{1+1}\cdot\begin{vmatrix} a & 1 \\ -1 & a \end{vmatrix} + 2\cdot(-1)^{1+3}\cdot\begin{vmatrix} 2 & a \\ 1 & -1 \end{vmatrix}$$

$$= a^2 + 1 + 2\cdot(-2-a) = a^2 + 1 - 4 - 2a = a^2 - 2a - 3$$

$$\det A = 0 \iff a^2 - 2a - 3 = 0$$

$$(a-3)(a+1) = 0$$

$$\underline{a=3} \vee \underline{a=-1}$$

A^{-1} existuje $\iff \det A \neq 0 \iff a \in \mathbb{R} \setminus \{3; -1\}$

$\underline{a=0}$:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} |\cdot(-2) \\ \oplus \\ (-1) \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \oplus \\ |\cdot(-1) \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & -1 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right) \oplus \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & -1 \\ 0 & 0 & -3 & -2 & 1 & 0 \end{array}\right) |:(-3)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & -1 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 \end{array}\right) \begin{array}{l} \oplus \\ (-2) \\ \oplus \end{array} \sim \underbrace{\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & -1 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 \end{array}\right)}_{A^{-1}}$$

Zkouška: $$A\cdot A^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & -1 \\ \frac{2}{3} & -\frac{1}{3} & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A\vec{x} = \vec{b} \implies \vec{x} = A^{-1}\vec{b} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & -1 \\ \frac{2}{3} & -\frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$

(2.) $B = \begin{pmatrix} 1 & 27 \\ 3 & 1 \end{pmatrix}$

λ je vlastním číslem matice $B \iff \det(B - \lambda E) = 0$

$$\det(B-\lambda E) = \begin{vmatrix} 1-\lambda & 27 \\ 3 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 27\cdot 3 = 0$$

$$(1-\lambda)^2 - 81 = 0$$
$$(1-\lambda)^2 - 9^2 = 0$$
$$(1-\lambda-9)(1-\lambda+9) = 0$$
$$(-8-\lambda)(10-\lambda) = 0$$
$$\underline{\lambda = -8} \vee \underline{\lambda = 10}$$

Vlastní vektory:

pro $\underline{\lambda_1 = -8}$: $\left(\begin{array}{cc|c} 9 & 27 & 0 \\ 3 & 9 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & 0 \end{array}\right)$ $\quad v_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

pro $\underline{\lambda_2 = 10}$: $\left(\begin{array}{cc|c} -9 & 27 & 0 \\ 3 & -9 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -3 & 0 \end{array}\right)$ $\quad v_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(3.) Matice, jejichž vlastní čísla jsou 7 a 8 jsou např.

$\begin{pmatrix} 7 & 0 \\ 0 & 8 \end{pmatrix}$ nebo $\begin{pmatrix} 7 & 1 \\ 0 & 8 \end{pmatrix}$