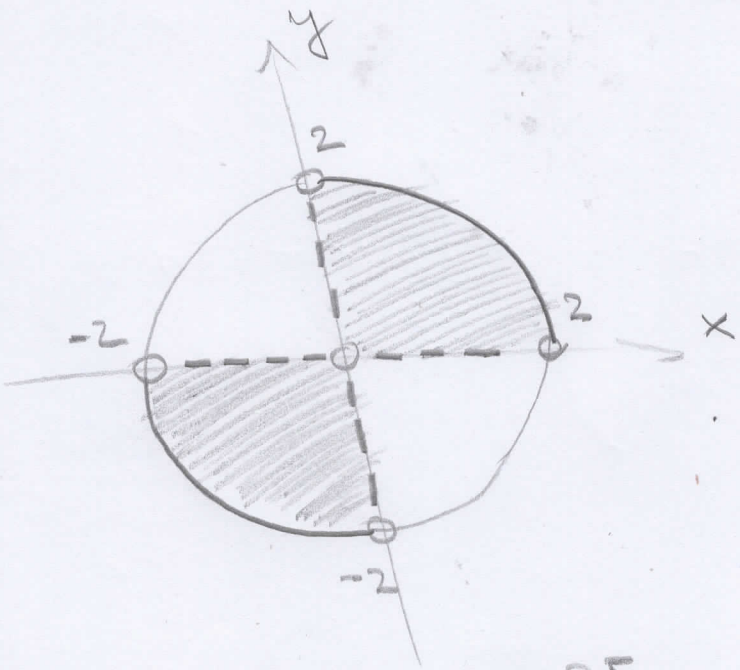
7. $F(x,y) = \left(\sqrt{4-x^2-y^2}, \log(xy)\right)$

$4-x^2-y^2 \geq 0 \quad \wedge \quad xy > 0$

$x^2+y^2 \leq 4$

$((x>0)\wedge(y>0)) \vee ((x<0)\wedge(y<0))$



$$\frac{\partial F_1}{\partial x} = \frac{-2x}{2\sqrt{4-x^2-y^2}} \qquad \frac{\partial F_2}{\partial x} = \frac{y}{xy} = \frac{1}{x}$$

$$\frac{\partial F_1}{\partial y} = \frac{-2y}{2\sqrt{4-x^2-y^2}} \qquad \frac{\partial F_2}{\partial y} = \frac{x}{xy} = \frac{1}{y}$$

$$J_F(x,y) = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} -\frac{x}{\sqrt{4-x^2-y^2}} & -\frac{y}{\sqrt{4-x^2-y^2}} \\ \frac{1}{x} & \frac{1}{y} \end{pmatrix}$$

$$J_F(1,1) = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & 1 \end{pmatrix}$$

(2.) $f(x,y) = g\left(\frac{x}{y}, \frac{y}{x}\right)$, $g \in C^2(\mathbb{R}^2)$

$g = g(a,b)$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial a} \cdot \frac{\partial a}{\partial y} + \frac{\partial g}{\partial b} \cdot \frac{\partial b}{\partial y}$$

$$= \frac{\partial g}{\partial a} \cdot \left(-\frac{x}{y^2}\right) + \frac{\partial g}{\partial b} \cdot \frac{1}{x}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(\frac{\partial^2 g}{\partial a^2} \cdot \left(-\frac{x}{y^2}\right) + \frac{\partial^2 g}{\partial a \partial b} \cdot \frac{1}{x}\right) \cdot \left(-\frac{x}{y^2}\right) +$$

$$+ \frac{\partial g}{\partial a} \cdot \frac{2x}{y^3} + \frac{1}{x} \cdot \left(\frac{\partial^2 g}{\partial b \partial a} \cdot \left(-\frac{x}{y^2}\right) +\right.$$

$$\left. + \frac{\partial^2 g}{\partial b^2} \cdot \frac{1}{x}\right)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \left(\frac{\partial^2 g}{\partial a^2}\left(\frac{1}{y}\right) + \frac{\partial^2 g}{\partial a \partial b} \cdot \left(-\frac{y}{x^2}\right)\right) \cdot \left(-\frac{x}{y^2}\right)$$

$$+ \frac{\partial g}{\partial a} \cdot \left(-\frac{1}{y^2}\right) + \left(\frac{\partial^2 g}{\partial b \partial a} \cdot \frac{1}{y} + \frac{\partial^2 g}{\partial b^2} \cdot \left(-\frac{y}{x^2}\right)\right) \cdot \frac{1}{x}$$

$$+ \frac{\partial g}{\partial x} \cdot \left(-\frac{1}{x^2}\right)$$

3. $f(x,y) = x^4 + y^3 + 2xy$

$z_0 = f(1,-1) = 1^4 + (-1)^3 + 2 \cdot 1 \cdot (-1) = \underline{\underline{-2}}$

Dotykový bod: $[1;-1;-2]$

$$\frac{\partial f}{\partial x} = 4x^3 + 2y \Big|_{[1;-1]} = \underline{\underline{2}}$$

$$\frac{\partial f}{\partial y} = 3y^2 + 2x \Big|_{[1;-1]} = \underline{\underline{5}}$$

Dotyková rovina: $z - z_0 = \frac{\partial f}{\partial x}(x_0,y_0)\cdot(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$

$$z+2 = 2(x-1) + 5(y+1)$$

$$z+2 = 2x - 2 + 5y + 5$$

$$2x + 5y - z + 1 = 0$$