1. $f(x,y) = y - x^2 \ln y \quad , \; A = [-1, 1]$

$$f(-1,1) = 1 - (-1)^2 \ln 1 = \underline{\underline{1}}$$

$$\frac{\partial f}{\partial x} = -2x \ln y \Big|_A = \underline{\underline{0}}$$

$$\frac{\partial f}{\partial y} = 1 - \frac{x^2}{y} \Big|_A = \underline{\underline{0}}$$

$$\frac{\partial^2 f}{\partial x^2} = -2 \ln y \Big|_A = \underline{\underline{0}}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{x^2}{y^2} \Big|_A = \underline{\underline{1}}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -\frac{2x}{y} \Big|_A = \underline{\underline{2}}$$

Taylorův polynom funkce f v bodě A řádu 2 je:

$$T(x,y) = f(A) + \frac{\partial f}{\partial x}(A)(x+1) + \frac{\partial f}{\partial y}(A)\cdot(y-1) +$$

$$+ \frac{1}{2!}\left(\frac{\partial^2 f}{\partial x^2}(A)(x+1)^2 + 2\cdot\frac{\partial^2 f}{\partial y \partial x}(A)(x+1)(y-1) + \frac{\partial^2 f}{\partial y^2}(A)(y-1)^2\right)$$

$$= 1 + 0 + 0 + \frac{1}{2}\left(0 + 2\cdot 2(x+1)(y-1) + 1\cdot(y-1)^2\right)$$

$$= 1 + 2xy + 2y - 2x - 2 + \frac{y^2}{2} - y + \frac{1}{2} = \underline{\underline{-\frac{1}{2} + \frac{y^2}{2} + y - 2x + 2xy}}$$

$$T(-1{,}1 \,;\, 1{,}1) \doteq \underline{\underline{0{,}985}}$$

(2.) $\overbrace{x^2+y^2-6y}^{=:F_1(x,y)}=0$

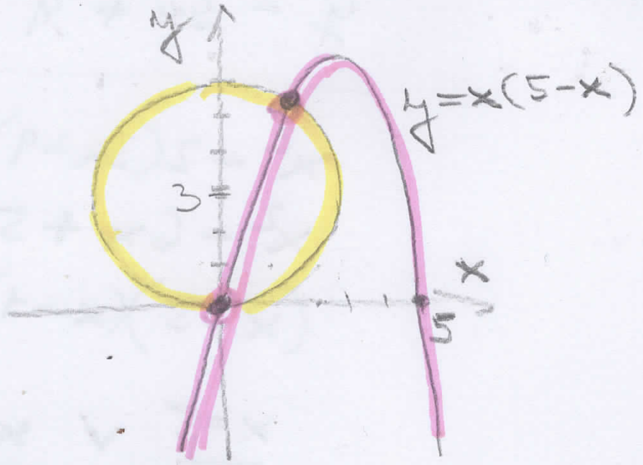
$x^2+y^2-6y+9=9$

$x^2+(y-3)^2=9$ ... kružnice se středem $[0;3]$ a poloměrem 3

$\overbrace{x^2-5x+y}^{=:F_2(x,y)}=0$

$y=5x-x^2=x(5-x)$ ... parabola

2 řešení



$$\left.\frac{\partial F_1}{\partial x}=2x\ \right|_{[2;5]}=4$$

$$\left.\frac{\partial F_1}{\partial y}=2y-6\ \right|_{[2,5]}=4$$

$$\left.\frac{\partial F_2}{\partial x}=2x-5\ \right|_{[2,5]}=-1$$

$$\frac{\partial F_2}{\partial y}=1$$

JAKOBIHO MATICE

$$J_F(2,5)=\begin{pmatrix}4&4\\-1&1\end{pmatrix}$$

$$\boxed{A^{-1}=\frac{1}{\det A}\cdot \operatorname{adj}A}$$

$$\begin{pmatrix}4&4\\-1&1\end{pmatrix}^{-1}=\frac{1}{8}\cdot\begin{pmatrix}1&-4\\1&4\end{pmatrix}$$

1. ITERACE: $\begin{pmatrix}x_1\\y_1\end{pmatrix}=\begin{pmatrix}x_0\\y_0\end{pmatrix}-J_F^{-1}(2,5)\cdot\begin{pmatrix}F_1(2,5)\\F_2(2,5)\end{pmatrix}$

$$=\begin{pmatrix}2\\5\end{pmatrix}-\frac{1}{8}\begin{pmatrix}1&-4\\1&4\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\end{pmatrix}\doteq\begin{pmatrix}1{,}625\\5{,}625\end{pmatrix}$$

(3.) $f(x,y) = x^3 + y^2 - 6xy - 39x + 18y + 20$

$$\frac{\partial f}{\partial x} = 3x^2 - 6y - 39 = 0 \quad |:3$$

$$\frac{\partial f}{\partial y} = 2y - 6x + 18 = 0 \quad |:2$$

$$x^2 - 2y - 13 = 0 \quad \leftarrow \text{dosazením}$$

$$y - 3x + 9 = 0 \quad \Longleftrightarrow \quad y = 3x - 9$$

$$x^2 - 2(3x-9) - 13 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$\frac{x=5}{y=6} \quad \vee \quad \frac{x=1}{y=-6}$$

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= 6x \\ \frac{\partial^2 f}{\partial y^2} &= 2 \\ \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} &= -6 \end{aligned}\right\} \quad \text{Hessova matice} \quad H(x,y) = \begin{pmatrix} 6x & -6 \\ -6 & 2 \end{pmatrix}$$

$$H(5,6) = \begin{pmatrix} 30 & -6 \\ -6 & 2 \end{pmatrix} \quad \det H = 60 - 36 > 0 \ \wedge \ \frac{\partial^2 f}{\partial x^2} > 0$$

$\Rightarrow$ [5; 6] je lok. min.

$$H(1,-6) = \begin{pmatrix} 6 & -6 \\ -6 & 2 \end{pmatrix} \quad \det H = 12 - 36 \leq 0$$

$\Rightarrow$ [1; -6] je sedlo