1.

x	1	2	5
y	1	0	4

$$\sum_{1}^{3} x_i^2 = 1^2 + 2^2 + 5^2 = 30$$

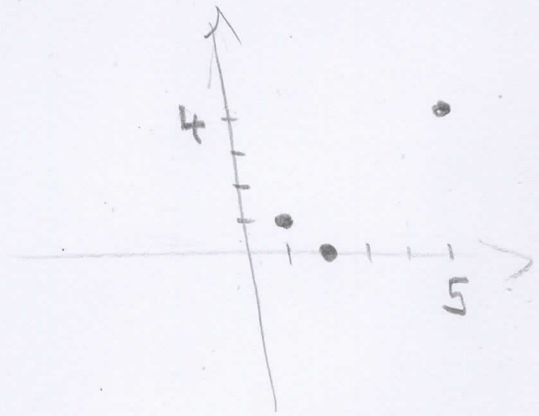
$$\sum_{1}^{3} x_i = 1 + 2 + 5 = 8$$

$$\sum_{1}^{3} y_i = 1 + 0 + 4 = 5$$

$$\sum_{1}^{3} x_i y_i = 1\cdot 1 + 2\cdot 0 + 5\cdot 4 = 21$$

$$\begin{pmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum x_i y_i \\ \sum y_i \end{pmatrix}$$

$$30a + 8b = 21 \quad | \cdot 3$$
$$8a + 3b = 5 \quad | \cdot (-8) \qquad 2 \oplus$$



$$26a + 0b = 23$$

$$a = \frac{23}{26} \doteq 0{,}88$$

$$8a + 3\cdot\frac{23}{26} = 5 \implies b = \frac{4}{13} \doteq 0{,}307$$

aproximace MNČ je $\boxed{y = \frac{23}{26}x + \frac{4}{13}}$

2.

$$\underbrace{e^{xy} - x^2 + y^3}_{=:\,F(x,y)} = 0$$

a) • $F \in C^1(\mathcal{O}(0,-1))$

• $F(0,-1) = e^0 - 0^2 + (-1)^3 = 0$

• $\frac{\partial F}{\partial y} = x e^{xy} + 3y^2 \Big|_{[0,-1]} = 3 \neq 0$

=> podle věty o implicitních funkcích rovnice $F(x,y) = 0$ definuje na okolí bodu $[0,-1]$ implicitní funkci $y = f(x) \in C^1$

b) $f(0) = -1$

$$f'(x) = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{y e^{xy} - 2x}{x e^{xy} + 3y^2}\Bigg|_{[0,-1]} = -\frac{-1}{3} = \frac{1}{3}$$

$$f''(x) = \left(-\frac{f(x)\cdot e^{xf(x)} - 2x}{x e^{xf(x)} + 3f^2(x)}\right)' = -$$

$$= -\frac{\left(f'(x)e^{xf(x)} + f(x)e^{xf(x)}\cdot(f(x) + xf'(x)) - 2\right)\left(x e^{xf(x)} + 3f^2(x)\right) - \left(f(x)e^{xf(x)} - 2x\right)\cdot\left(e^{xf(x)} + x e^{xf(x)}\cdot(f(x) + xf'(x)) + 6f(x)\cdot f'(x)\right)}{\left(x e^{xf(x)} + 3f^2(x)\right)^2}$$

$$\Rightarrow f''(0) = \left(-\frac{1}{9}\right)\cdot\left(\left(\frac{1}{3} - 1(-1+0) - 2\right)(0+3) - (-1-0)\left(1 + 6(-1)\cdot\frac{1}{3}\right)\right)$$

$$= \left(-\frac{1}{9}\right)(-2-1) = \frac{3}{9} = \underline{\underline{\frac{1}{3}}}$$

Taylorův polynom funkce f v bodě $[0;-1]$

je $T^2_{f,0}(x) = f(0) + f'(0)\cdot(x-0) + \frac{f''(0)}{2!}(x-0)^2$

$$= -1 + \frac{1}{3}x + \frac{1}{6}x^2$$

3. $\underbrace{x^3 + 3xyz - 8}_{=F(x,y,z)} = 0 \qquad x = f(y,z)$

$$\frac{\partial f}{\partial z} = -\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial x}} = -\frac{3xy}{3x^2+3yz}$$

$$\frac{\partial^2 f}{\partial z \partial y} = \frac{\partial}{\partial y}\left(-\frac{3f(y,z)y}{3f^2(y,z)+3yz}\right) =$$

$$= -\frac{\left(3\frac{\partial f}{\partial y}\cdot y + 3f(y,z)\right)\left(3f^2(y,z)+3yz\right) - \left(3f(y,z)y\cdot\right.}{\left(3f^2(y,z)+3yz\right)^2}$$

$$\frac{\left.\cdot\left(6f(y,z)\frac{\partial f}{\partial y} + 3z\right)\right)}{}$$