2\. ZT, Varianta A

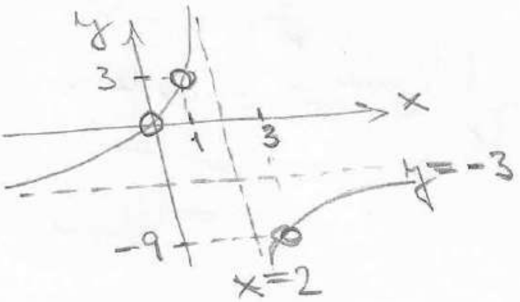
1\. $f(x) = \left(\frac{3}{x-2} - \frac{6}{x-1}\right) : \left(\frac{1}{x} - \frac{2}{x^2-x}\right) =$

$= \frac{3(x-1)-6(x-2)}{(x-2)(x-1)} : \frac{x-1-2}{x(x-1)} = \frac{3x-3-6x+12}{(x-2)(x-1)} : \frac{x-3}{x(x-1)} =$

$= \frac{-3x+9}{(x-2)\cancel{(x-1)}} \cdot \frac{x\cancel{(x-1)}}{x-3} = \frac{(-3)\cancel{(x-3)}\cdot x}{(x-2)\cancel{(x-3)}} = \frac{-3x}{x-2}, \; x \neq 0, 1, 2, 3$

$D_f = \mathbb{R} \setminus \{0, 1, 2, 3\}$

$H_f = \mathbb{R} \setminus \{0, 3, -3, -9\}$



2\. $f(x) = \sqrt{\frac{2x-3x^2}{5+6x}}$

$\frac{2x-3x^2}{5+6x} = \frac{x(2-3x)}{5+6x} \geq 0$

$+ \quad - \quad + \quad -$

$-\frac{5}{6} \quad 0 \quad \frac{2}{3}$

$\boxed{D_f = \left(-\infty, -\frac{5}{6}\right) \cup \left\langle 0, \frac{2}{3} \right\rangle}$

3\. $f: y = \log_2\left(\frac{4x+1}{3x+1}\right)$

$x = \log_2\left(\frac{4y+1}{3y+1}\right)$

$2^x = \frac{4y+1}{3y+1}$

$2^x \cdot 3y + 2^x = 4y + 1$

$2^x \cdot 3y - 4y = 1 - 2^x$

$(3 \cdot 2^x - 4)y = 1 - 2^x$

$\boxed{f^{-1}: \; y = \frac{1-2^x}{3 \cdot 2^x - 4}}, \; x \neq \log_2 \frac{4}{3}$

4. a) $$\lim_{x\to+\infty}\frac{2^{3x-1}+(\sqrt{3})^{4x+1}}{2^{3x+1}+(\sqrt{3})^{4x-3}}=\lim_{x\to\infty}\frac{\frac{1}{2}\cdot 8^x+\sqrt{3}\cdot 9^x}{2\cdot 8^x+(\sqrt{3})^{-3}\cdot 9^x}$$

$$=\lim_{x\to+\infty}\frac{\cancel{9^x}\cdot\left(\frac{1}{2}\cdot\left(\frac{8}{9}\right)^x+\sqrt{3}\right)}{\cancel{9^x}\cdot\left(2\cdot\left(\frac{8}{9}\right)^x+(\sqrt{3})^{-3}\right)}=\frac{0+\sqrt{3}}{0+(\sqrt{3})^{-3}}=(\sqrt{3})^4=\underline{\underline{9}}$$

b) $$\lim_{x\to\infty}\log_{\sqrt{8}}\left(\sqrt{x^2+4x}-x\right)=\log_{\sqrt{8}}\left(\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)\cdot\frac{\sqrt{x^2+4x}+x}{\sqrt{x^2+4x}+x}\right)$$

$$=\log_{\sqrt{8}}\left(\underbrace{\lim_{x\to\infty}\frac{\cancel{x^2}+4x-\cancel{x^2}}{\sqrt{x^2+4x}+x}}\right)=\log_{\sqrt{8}}2=\underline{\underline{\frac{1}{3}}}$$

$$=\lim_{x\to\infty}\frac{4\cancel{x}}{\cancel{x}\left(\sqrt{1+\frac{4}{x}}+1\right)}=\frac{4}{\sqrt{1+0}+1}=2$$

c) $$\lim_{x\to0}\frac{3x+xe^{3x}+1-e^{2x}}{x^3+6x^2+5x}\overset{\text{L.P.}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}}\lim_{x\to0}\frac{3+e^{3x}+xe^{3x}\cdot3-2e^{2x}}{3x^2+12x+5}=\underline{\underline{\frac{2}{5}}}$$

5. $$\left(\ln\left(\frac{5x+1}{5x-1}\right)\right)'=\frac{1}{\frac{5x+1}{\cancel{5x-1}}}\cdot\frac{5(5x-1)-(5x+1)\cdot5}{(5x-1)^{\cancel{2}}}=$$

$$=\frac{25x-5-25x-5}{(5x+1)(5x-1)}=\underline{\underline{-\frac{10}{25x^2-1}}}$$

6. $$f(x)=\cos(3x)+e^{-x}\Big|_{x=0}=2$$

$$f'(x)=-3\sin(3x)-e^{-x}\Big|_{x=0}=-1$$

$$f''(x)=-9\cos(3x)+e^{-x}\Big|_{x=0}=-8$$

$$T^2_{f,0}(x)=2+(-1)x+\frac{(-8)}{2!}x^2=\underline{\underline{2-x-4x^2}}$$

7.

$$f(x)=\sqrt{x}\,\Big|_{x=4}=2$$

$$f'(x)=\frac{1}{2\sqrt{x}}\,\Big|_{x=4}=\frac{1}{4}$$

$$T^{1}_{\sqrt{x},4}(x)=2+\frac{1}{4}(x-4)$$

$$\sqrt{3{,}98}\doteq 2+\frac{1}{4}(3{,}98-4)=2-\frac{1}{4}\cdot 0{,}02=2-0{,}005=\underline{\underline{1{,}995}}$$

8.

$$f(x)=\frac{3x-2}{2x^2}$$

$$f'(x)=\frac{3\cdot 2x^2-(3x-2)\cdot 4x}{4x^4}=\frac{6x^2-12x^2+8x}{4x^4}=\frac{-6x^2+8x}{4x^4}=\frac{-3x+4}{2x^3}$$

$$f'(x)=0 \iff x=\frac{4}{3}$$

$$\searrow \qquad \nearrow \qquad \searrow$$

$$f'<0 \quad 0 \quad f'>0 \quad \frac{4}{3} \quad f'<0$$

$$f\left(\frac{4}{3}\right)=\frac{3\cdot\frac{4}{3}-2}{2\cdot\left(\frac{4}{3}\right)^2}=\frac{9}{16} \qquad \text{lok. max. } \left[\frac{4}{3};\frac{9}{16}\right]$$

$$f''(x)=\frac{(-3)\cdot 2x^3-(-3x+4)\cdot 6x^2}{4x^6}=\frac{\cancel{2x^2}\cdot(-3x+9x-12)}{\cancel{4x^6}}=\frac{6x-12}{2x^4}$$

$$f''(x)=0 \iff x=2$$

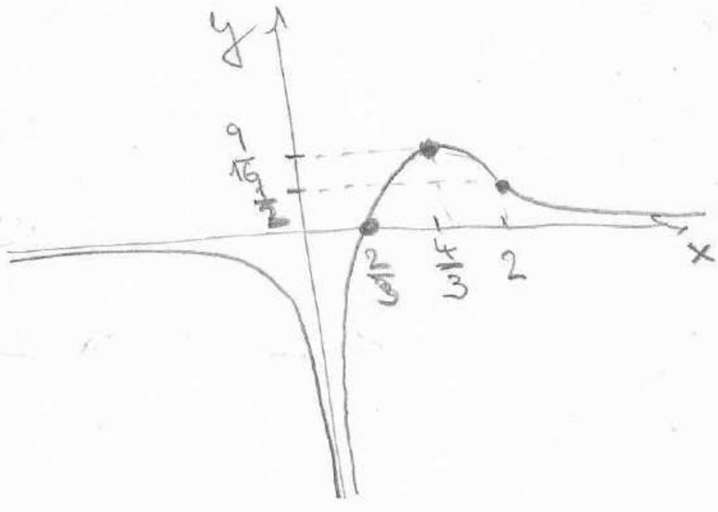
$$\cap \qquad \cap \qquad \cup$$

$$f''<0 \quad 0 \quad f''<0 \quad 2 \quad f''>0$$

$$f(2)=\frac{4}{2\cdot 4}=\frac{1}{2} \qquad \text{inflexní bod } \left[2;\frac{1}{2}\right]$$

$$\lim_{x\to 0} f(x)=\frac{-2}{0_+}=-\infty$$

$$\lim_{x\to\pm\infty} f(x)\overset{\text{L.P.}}{=}\lim_{x\to\pm\infty}\frac{3}{4x}=0$$



$$H_f=\left(-\infty,\frac{9}{16}\right\rangle$$