1.ZT, varianta B

1) $\neg\left(\forall x\in\mathbb{R}\ \exists y\in\mathbb{R}:\ x^2+y^2\leq 4\right)$

$\Longleftrightarrow\ \exists x\in\mathbb{R}\ \forall y\in\mathbb{R}:\ x^2+y^2>4$

2) $AXB - A = 1$

$AXB = 1 + A \quad |\cdot B^{-1}$ zprava

$AX = (1+A)\cdot B^{-1} \quad |\cdot A^{-1}$ zleva

$X = A^{-1}\cdot(1+A)\cdot B^{-1}$

$$A^{-1} = \frac{1}{\det A}\cdot \operatorname{adj}A = \frac{1}{-1}\cdot\begin{pmatrix}-1 & 1\\ -1 & 2\end{pmatrix} = \begin{pmatrix}1 & -1\\ 1 & -2\end{pmatrix}$$

$$B^{-1} = \frac{1}{\det B}\cdot \operatorname{adj}B = \frac{1}{10-12}\cdot\begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix} = \begin{pmatrix}-1 & -\frac{3}{2}\\ -2 & -\frac{5}{2}\end{pmatrix}$$

$$A+1 = \begin{pmatrix}2 & -1\\ 1 & -1\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix}$$

$$X = \underbrace{\begin{pmatrix}1 & -1\\ 1 & -2\end{pmatrix}\cdot\begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix}}_{\begin{pmatrix}2 & -1\\ 1 & -1\end{pmatrix}}\cdot\begin{pmatrix}-1 & -\frac{3}{2}\\ -2 & -\frac{5}{2}\end{pmatrix} = \underline{\underline{\begin{pmatrix}0 & -\frac{1}{2}\\ 1 & 1\end{pmatrix}}}$$

Zkouška: $$AXB - A = \underbrace{\begin{pmatrix}2 & -1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0 & -\frac{1}{2}\\ 1 & 1\end{pmatrix}}_{\begin{pmatrix}-1 & -2\\ -1 & -\frac{3}{2}\end{pmatrix}}\begin{pmatrix}5 & -3\\ -4 & 2\end{pmatrix} - \begin{pmatrix}2 & -1\\ 1 & -1\end{pmatrix} =$$

$$= \begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix} - \begin{pmatrix}2 & -1\\ 1 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 1$$

3) $A = \begin{pmatrix} 1 & 3 & 16 \\ 0 & 1 & 12 \\ 0 & 0 & 4 \end{pmatrix}$

$\det A = 4 \neq 0 \Rightarrow$ matice je regulární

$h(A) = 3$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 16 & 1 & 0 & 0 \\ 0 & 1 & 12 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \cdot(-4) \\ \cdot(-3) \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & -4 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \cdot(-3) \\ :4 \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 5 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{4} \end{array}\right)$$

$\underbrace{\quad}_{A^{-1}}$

Matice A je regulární, tedy má lin. nezávislé řádky, proto 3. řádek nelze zapsat jako lin. kombinace ostatních.

$B = \begin{pmatrix} 1 & 4 & 5 \\ 1 & 3 & 4 \\ 1 & -2 & -1 \end{pmatrix}$

Matice B je singulární, neboť 3. sloupec je součtem prvních dvou: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$,

tedy sloupcové vektory jsou lineárně závislé, proto $\det B = 0$ a B^{-1} neexistuje.

$\exists a, b:$ $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \stackrel{?}{=} a \cdot \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} + b \cdot \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$? $\Rightarrow$

$1 = a + b$

$-2 = 4a + 3b$

$-1 = 5a + 4b$

$a = 1 - b$

$-2 = 4(1-b) + 3b$

$-2 = 4 - 4b + 3b$

$-6 = -b$

$b = 6$

$a = -5$

Ano, lze:

$$\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} = (-5) \cdot \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} + 6 \cdot \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

4) $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 2 & 4 & -1 \end{pmatrix}$

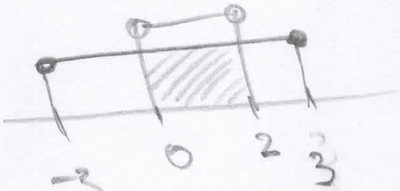
a) $A\vec{x} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 2 & 4 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\cdot\vec{x}$ ANO, $\lambda = 0$

b) $A\vec{x} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 2 & 4 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} = 3\cdot\vec{x}$ ANO, $\lambda = 3$

c) $A\vec{x} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 2 & 4 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \neq \lambda\cdot\vec{x}$ NE

5) $f(x) = \sqrt{9-x^2} + \log(2x-x^2)$

I. $9-x^2 \geq 0 \iff x \in \langle -3, 3 \rangle$



II. $2x - x^2 > 0$

$x(2-x) > 0 \iff x \in (0, 2)$

$D_f = (0, 2)$