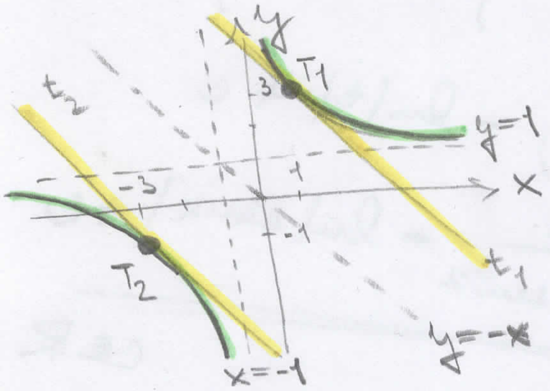
(A)

1. $f(x) = \frac{x+5}{x+1}$ $D_f = R - \{-1\}$

a) $f'(x) = \frac{x+1-(x+5)}{(x+1)^2} = -\frac{4}{(x+1)^2} \overset{?}{=} -1$



$4 = (x+1)^2$
$4 = x^2 + 2x + 1$
$0 = x^2 + 2x - 3$
$0 = (x+3)(x-1)$
$x = 1 \vee x = -3$
$f(1) = 3$ $f(-3) = -1$

dotyčné body: $T_1 = [1;3]$
$T_2 = [-3;-1]$

dotyčnice: $t_1: y = 3 + (-1)\cdot(x-1) = -x+4$
$t_2: y = -1 + (-1)\cdot(x+3) = -x-4$

b) $f''(x) = \frac{8}{(x+1)^3}$

$f''(1) = 1$

$f''(-3) = -1$

TAYLOROVY POLYNOMY 2. RÁDU

$g_1: y = 3 + (-1)(x-1) + \frac{1}{2!}\cdot(x-1)^2$

$g_2: y = -1 + (-1)(x+3) + \frac{(-1)}{2!}(x+3)^2$

c) Diferenciál: $df(1, dx) = (-1)\cdot dx$
$df(-3, dx) = (-1)\, dx$

d) $\lim_{x \to +\infty} \frac{x+5}{x+1} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{L.P.}{=}} \lim_{x \to \infty} \frac{1}{1} = 1$

(2)

$$\int \operatorname{ctg}^3 x\,dx = \int \frac{\cos^3 x}{\sin^3 x}\,dx \overset{\sin^2 x+\cos^2 x=1}{=} \int \frac{1-\sin^2 x}{\sin^3 x}\cdot \cos x\,dx$$

$$= \left| \begin{matrix} \sin x = t \\ \cos x\,dx = dt \end{matrix} \right| \int \frac{1-t^2}{t^3}\,dt = \int t^{-3}\,dt - \int \frac{1}{t}\,dt =$$

$$= \frac{t^{-2}}{(-2)} - \ln|t| + c$$

$$= -\frac{1}{2\sin^2 x} - \ln|\sin x| + c$$

$c \in \mathbb{R}$

$$\int_0^1 \sqrt[5]{x^2 \cdot \sqrt[4]{\sqrt{x}\cdot x^3}}\,dx = \int_0^1 \left(x^2 \underbrace{\left(x^{\frac{1}{2}}\cdot x^3\right)^{\frac{1}{4}}}_{x^{\frac{7}{8}}}\right)^{\frac{1}{5}} dx =$$

$$= \int_0^1 \left(\underbrace{x^2 \cdot x^{\frac{1}{2}}}_{x^{\frac{5}{2}}}\right)^{\frac{1}{5}} dx = \int_0^1 \left(x^{\frac{5}{2}}\right)^{\frac{1}{5}} dx = \int_0^1 x^{\frac{1}{2}}\,dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\cdot 1^{\frac{3}{2}} - \frac{2}{3}\cdot 0^{\frac{3}{2}} = \frac{2}{3}$$