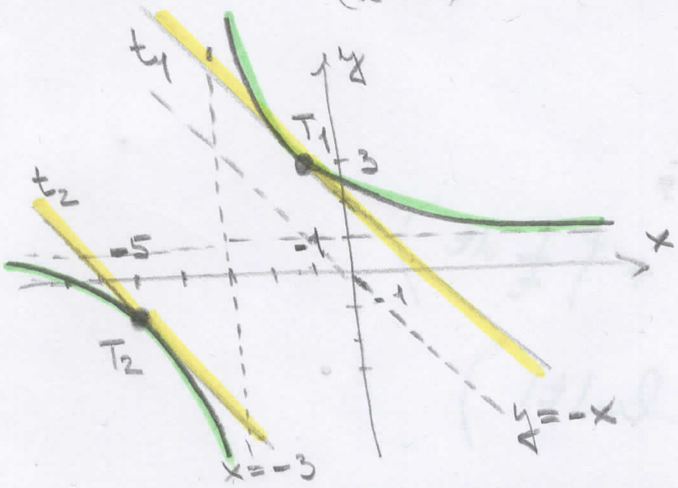
B

1. $f(x) = \dfrac{x+7}{x+3}$

a) $f'(x) = \dfrac{x+3-(x+7)}{(x+3)^2} = -\dfrac{4}{(x+3)^2} \overset{?}{=} -1$



$4 = (x+3)^2$

$0 = (x+3)^2 - 4$

$0 = (x+3-2)(x+3+2)$

$0 = (x+1)(x+5)$

$x=-1 \quad \vee \quad x=-5$

$f(-1) = 3 \qquad f(-5) = -1$

tečné body: $[-1;-3] = T_1$

$[-5,-1] = T_2$

tečny: $t_1: y = 3 + (-1)\cdot(x+1) = -x+2$

$t_2: y = -1 + (-1)(x+5) = -x-6$

b) $f''(x) = \dfrac{8}{(x+3)^3}$

$f''(-1) = 1$

$f''(-5) = -1$

TAYLOROVY POLYNOMY 2. ŘÁDU

$g_1: y = 3 + (-1)(x+1) + \frac{1}{2!}\cdot(x+1)^2$

$g_2: y = -1 + (-1)(x+5) + \frac{(-1)}{2!}(x+5)^2$

c) Diferenciál: $df(-1, dx) = (-1)\,dx$

$df(-3, dx) = (-1)\,dx$

d) $\lim\limits_{x\to\infty} \dfrac{x+7}{x+3} \overset{L.P.}{\underset{\text{„}\frac{\infty}{\infty}\text{“}}{=}} \lim\limits_{x\to\infty} \dfrac{1}{1} = \underline{\underline{1}}$

(2.)

$\sin^2 x + \cos^2 x = 1$

$$\int tg^5 x\, dx = \int \frac{\sin^5 x}{\cos^5 x}\, dx = \int \frac{(1-\cos^2 x)^2}{\cos^5 x} \cdot \sin x\, dx$$

$$= -\int \frac{(1-t^2)^2}{t^5}\, dt = -\int \frac{1-2t^2+t^4}{t^5}\, dt =$$

$$\left|\begin{array}{l} \cos x = t \\ -\sin x\, dx = dt \end{array}\right|$$

$$= -\left(\int t^{-5} dt - 2\cdot\int t^{-3} dt + \int \frac{1}{t} dt\right)$$

$$= -\left(\frac{t^{-4}}{(-4)} - \cancel{2}\cdot\frac{t^{-2}}{(\cancel{-2})} + \ln|t|\right)$$

$$= \frac{1}{4t^4} - \frac{1}{t^2} - \ln|t| + c$$

$$= \frac{1}{4\cos^4 x} - \frac{1}{\cos^2 x} - \ln|\cos x| + c,\ c \in \mathbb{R}$$

$$\int_0^1 \sqrt[7]{x^3 \cdot \sqrt[5]{x^2 \cdot \sqrt{x}}}\, dx = \int_0^1 \sqrt{x}\, dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}$$

$$\sqrt[7]{x^3 \cdot \sqrt[5]{x^2 \cdot \sqrt{x}}} = \left(x^3 \cdot \underbrace{\left(x^2 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{5}}}_{x^{\frac{5}{2}}}\right)^{\frac{1}{7}} = \left(\underbrace{x^3 \cdot x^{\frac{1}{2}}}_{x^{\frac{7}{2}}}\right)^{\frac{1}{7}} = x^{\frac{1}{2}}$$