X … počet neúspěchů předcházejících 3. úspěchu

přičemž $P(\text{úspěch}) = \frac{1}{6}$

$$P[X=k] = \binom{k+2}{k} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^k \quad , k \in \mathbb{N}_0$$

Ověřme, že $\sum\limits_{k=0}^{\infty} P[X=k] = 1$.

$$\sum_{k=0}^{\infty} P[X=k] = \sum_{k=0}^{\infty} \underbrace{\frac{(k+2)!}{k! \cdot (k+2-k)!}}_{\frac{(k+2)(k+1)\cdot \cancel{k!}}{2\cdot \cancel{k!}}} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^k =$$

$$= \left(\frac{1}{6}\right)^3 \cdot \frac{1}{2} \cdot \sum_{k=0}^{\infty} \underbrace{(k+2)(k+1) \cdot \left(\frac{5}{6}\right)^k}_{\left(x^{k+2}\right)'' \Big|_{x=\frac{5}{6}}}$$

$$\boxed{x = \frac{5}{6}} \; = \left(\frac{1}{6}\right)^3 \cdot \frac{1}{2} \cdot \left(\sum_{k=0}^{\infty} x^{k+2}\right)'' = \left(\frac{1}{6}\right)^3 \cdot \frac{1}{2} \cdot \left(\frac{x^2}{1-x}\right)'' \Bigg|_{x=\frac{5}{6}}$$

$$= \left(\frac{1}{6}\right)^3 \cdot \frac{1}{2} \cdot \frac{2}{(1-x)^3} \Bigg|_{x=\frac{5}{6}} = \frac{1}{6^3} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\frac{1}{6^3}} = \underline{\underline{1}}$$

$$\left(\frac{x^2}{1-x}\right)'' = \left(\frac{2x(1-x) - (-1)x^2}{(1-x)^2}\right)' = \left(\frac{2x - x^2}{(1-x)^2}\right)' = \frac{(2-2x)(1-x)^2 + (2x-x^2)\cdot 2(1-x)}{(1-x)^4}$$