$f(x) = x \ln(x^2) = 2x \ln x \quad ; D_f = \mathbb{R} \setminus \{0\}$

$f(-x) = (-x) \cdot \ln((-x)^2) = -x \ln(x^2) = -f(x) \Rightarrow$ Funkce je <u>lichá</u>

nebot' $\forall x \in D_f : f(-x) = -f(x)$

<u>1. derivace</u>: $f'(x) = \left(x \ln(x^2)\right)' = 1 \cdot \ln(x^2) + x \cdot 2x \cdot \frac{1}{x^2} =$

$= \ln x^2 + 2$ $\qquad \boxed{(a \cdot b)' = a' \cdot b + a \cdot b'}$

$f'(x) = 0 \iff$

$2 \ln x + 2 = 0 \quad | :2$

$\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1}$

$f'>0 \quad -\frac{1}{e} \quad f'<0 \quad 0 \quad f'<0 \quad \frac{1}{e} \quad f'>0$

$\forall x \in (-\infty, -\frac{1}{e}) \cup (\frac{1}{e}, +\infty) : f'(x) > 0 \Rightarrow f$ <u>roste</u>

$\forall x \in (-\frac{1}{e}, 0) \cup (0, \frac{1}{e}) : f'(x) < 0 \Rightarrow f$ <u>klesá</u>

$f(\frac{1}{e}) = \frac{1}{e} \cdot \underbrace{\ln\left(\frac{1}{e^2}\right)}_{=-2} = -\frac{2}{e} \Rightarrow$ lok. minimum: $[\frac{1}{e}; -\frac{2}{e}]$

lok. maximum: $[-\frac{1}{e}; \frac{2}{e}]$

<u>2. derivace</u>: $f''(x) = \frac{2}{x}$ $\qquad f''<0 \quad 0 \quad f''>0$

$\forall x \in (0, \infty) : f''(x) > 0 \Rightarrow f$ je <u>konvexní</u>

$\forall x \in (-\infty, 0) : f''(x) < 0 \Rightarrow f$ je <u>konkávní</u>

průsečíky s osami: $0 \notin D_f \Rightarrow f$ nemá průsečík s osou y

$f(x) = 0 \iff x \ln(x^2) = 0$

$x = 0 \vee \ln(x^2) = 0$

$\notin D_f \qquad x = \pm 1$

$P_{x_1} = [1; 0]$

$P_{x_2} = [-1; 0]$

limity: $\lim\limits_{x \to +\infty} x \ln x^2 = \underline{\underline{+\infty}}$
$\lim\limits_{x \to -\infty} x \ln x^2 = \underline{\underline{-\infty}}$ $\Bigg\}$ $\Rightarrow$ nemá vodorovnou asymptotu

$$\lim_{x \to 0} x \ln x^2 = \lim_{x \to 0} \frac{2 \ln x}{\frac{1}{x}} \overset{L.P.}{=} \lim_{x \to 0} \frac{2 \cdot \frac{1}{x}}{-\frac{1}{x^2}} =$$

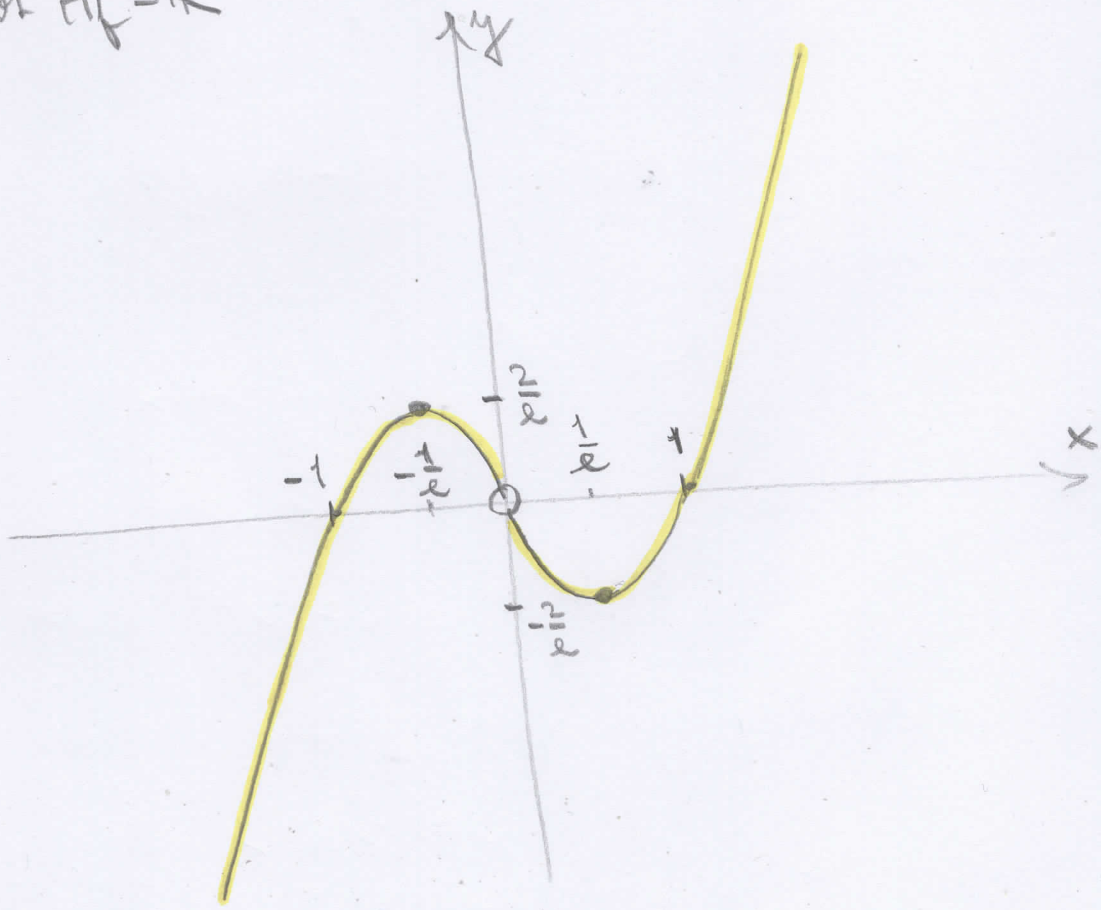
$$\text{"}\frac{\infty}{\infty}\text{"}$$

$$= \lim_{x \to 0} \left(-\frac{2}{x} \cdot \frac{x^2}{1} \right) = -\lim_{x \to 0} 2x = \underline{\underline{0}}$$

šikmá asymptota: $k = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \frac{\cancel{x} \ln x^2}{\cancel{x}} = +\infty$

$\Rightarrow$ nemá šikmou asymptotu

Obor hodnot $H_f = \mathbb{R}$



$f(x) = \dfrac{2x}{(x-1)^2}$

x	-1	1	-2	0
$f(x)$	$-\frac{1}{2}$	N.D.	$-\frac{4}{9}$	0

$D_f = \mathbb{R} \setminus \{1\}$

DERIVACE PODÍLU $\left(\dfrac{a}{b}\right)' = \dfrac{a' \cdot b - a \cdot b'}{b^2}$

1. derivace: $f'(x) = \dfrac{2\cdot(x-1)^2 - 2x \cdot 2(x-1)}{(x-1)^4} = \dfrac{\cancel{(x-1)}\cdot\left(2(x-1) - 4x\right)}{(x-1)^{\cancel{4}\,3}}$

$= \dfrac{-2x-2}{(x-1)^3} = (-2)\cdot\dfrac{x+1}{(x-1)^3}$

		-1		1	
$(-2)(x+1)$	$+$	•	$-$		$-$
$(x-1)^3$	$-$		$-$	∘	$+$
	$-\infty$ $-$		$+$		$-$ ∞

$f'<0$ $\quad -1 \quad$ $f'>0$ $\quad 1 \quad$ $f'<0$

$f'(x) = 0 \iff x = -1$

$\forall x \in (-1,1): f'(x) > 0 \implies$ Funkce f je rostoucí na $(-1,1)$

$\forall x \in (-\infty,-1) \cup (1,\infty): f'(x) < 0 \implies$ Funkce f je klesající na $(-\infty,-1)$ a na $(1,\infty)$

$f(-1) = \dfrac{2\cdot(-1)}{(-1-1)^2} = -\dfrac{2}{4} = -\dfrac{1}{2} \implies$ Lokální minimum $[-1; -\frac{1}{2}]$

2. derivace: $f''(x) = \left(\dfrac{-2x-2}{(x-1)^3}\right)' = \dfrac{(-2)\cdot(x-1)^3 - (-2x-2)\,3(x-1)^2}{(x-1)^6} =$

$= \dfrac{\cancel{(x-1)^2}\cdot\left(-2(x-1) + (2x+2)\cdot 3\right)}{(x-1)^{\cancel{6}\,4}} = \dfrac{-2x+2+6x+6}{(x-1)^4} = \dfrac{4x+8}{(x-1)^4}$

$f''(x) = 0 \iff 4x + 8 = 0 \quad | :4$

$x + 2 = 0 \quad | -2$

$x = -2$

$f(-2) = \dfrac{2\cdot(-2)}{(-2-1)^2} = -\dfrac{4}{9}$

Inflexní bod: $[-2; -\frac{4}{9}]$

$f''<0$ $\quad -2 \quad$ $f''>0$ $\quad 1 \quad$ $f''>0$

$\forall x \in (-\infty,-2): f''(x) < 0$

$\implies f$ je konkávní na $(-\infty,-2)$

$\forall x \in (-2,1) \cup (1,\infty): f''(x) > 0 \implies f$ je konvexní na $(-2,1)$ a $(1,\infty)$

průsečíky s osami: $f(0) = \frac{2 \cdot 0}{(0-1)^2} = 0$

$$\frac{2x}{(x-1)^2} = 0 \iff x = 0$$

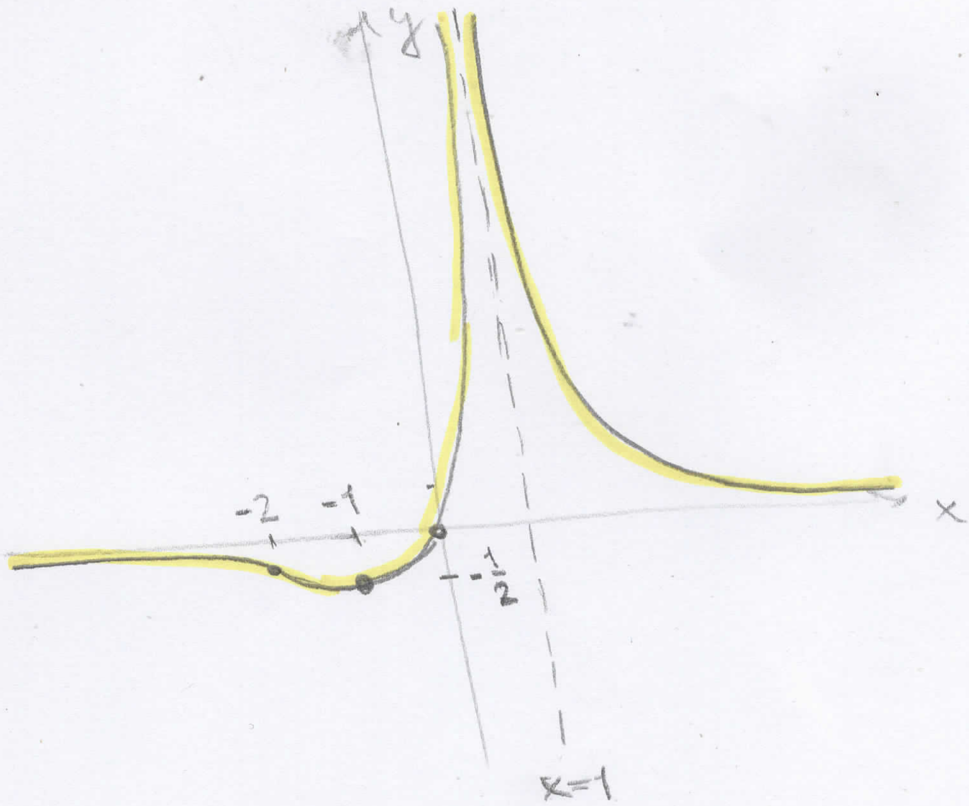
$P_x = P_y = [0;0]$

$$\lim_{x \to \pm\infty} \frac{2x}{(x-1)^2} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{L.P.}{=}} \lim_{x \to \pm\infty} \frac{2}{2(x-1)} = \frac{2}{\infty} = \underline{\underline{0}}$$

$\Rightarrow$ přímka $y = 0$ je vodorovná asymptota v $\pm\infty$

$$\lim_{x \to 1} \frac{2x}{(x-1)^2} = \frac{2}{0_+} = \underline{\underline{+\infty}}$$

$\Rightarrow$ přímka $x = 1$ je svislá asymptota



Obor hodnot $H_f = \langle -\frac{1}{2}, +\infty)$