(1.) $\begin{vmatrix} -2 & a & 0 \\ 0 & 5 & 1 \\ a & 12 & a \end{vmatrix} = -10a + 0 + a^2 - 0 + 24 + 0$

$= a^2 - 10a + 24 = (a-6)(a-4) \neq 0$

Matice je regulární $\iff \det A \neq 0 \iff a \in \mathbb{R} \setminus \{4; 6\}$

(2.) $\begin{pmatrix} 3 & 7 \\ 5 & 12 \end{pmatrix}^{-1} = \frac{1}{36-35} \cdot \begin{pmatrix} 12 & -7 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 12 & -7 \\ -5 & 3 \end{pmatrix}$

$\boxed{A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A}$

(3.) $\begin{vmatrix} 7-\lambda & 4 \\ 1 & 7-\lambda \end{vmatrix} = (7-\lambda)^2 - 4 = (7-\lambda-2)(7-\lambda+2)$

$= (5-\lambda)(9-\lambda) = 0$

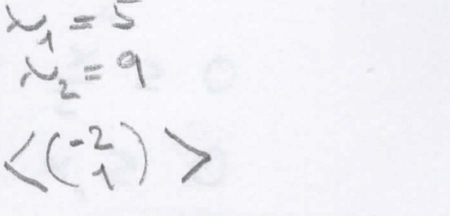
$\iff \lambda_1 = 5$
$\lambda_2 = 9$

Vlastní vektory: $M_5 = \operatorname{Ker}\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} = \left\langle \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\rangle$

$M_9 = \operatorname{Ker}\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix} = \left\langle \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle$

(4.) $f(x) = \frac{\sqrt{2x-3}}{x^2 - 2x - 24} + \log(56 - 7x)$

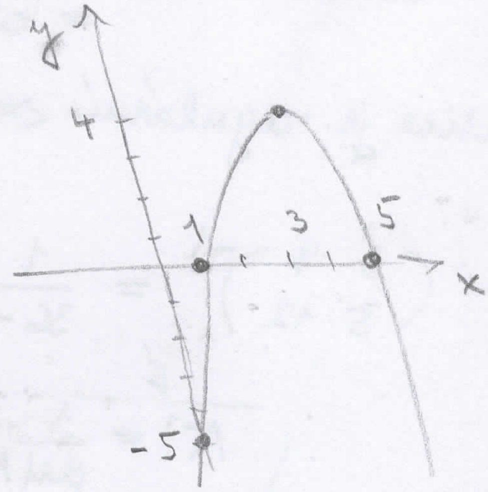
- $2x - 3 \geq 0$
  $x \geq \frac{3}{2}$
- $x^2 - 2x - 24 \neq 0$
  $(x-6)(x+4) \neq 0$
  $x \neq 6 \ \wedge \ x \neq -4$
- $56 - 7x > 0$
  $56 > 7x$
  $x < 8$




$D = \langle \frac{3}{2}, 6) \cup (6, 8)$

5. $f(x) = -x^2+6x-5 = (-1)(x^2-6x+5) = (-1)(x-5)(x-1)$

| $x$ | $0$ | $5$ | $1$ | $3$ |
|---|---|---|---|---|
| $f(x)$ | $-5$ | $0$ | $0$ | $4$ |




6. $f(x) = 2\arccos\left(\frac{x}{5}-1\right)+\pi$

$-1 \le \frac{x}{5}-1 \le 1 \quad |+1$

$0 \le \frac{x}{5} \le 2 \quad |\cdot 5$

$0 \le x \le 10$

$\boxed{D = \langle 0, 10\rangle}$

$H_{\arccos} = \langle 0, \pi\rangle$

$H_{2\arccos} = \langle 0, 2\pi\rangle$

$\boxed{H = \langle \pi, 3\pi\rangle}$

7. $\lim\limits_{x\to 2} \frac{x^3+3x^2-10x}{7x^2-7x-14} = \lim\limits_{x\to 2} \frac{x(x+5)\cancel{(x-2)}}{7\cancel{(x-2)}(x+1)} = \frac{2\cdot 7}{7\cdot 3} = \underline{\underline{\frac{2}{3}}}$

$\lim\limits_{x\to\infty} \frac{x+2x^3-24x^4}{1+3x^4+x^3} = \lim\limits_{x\to\infty} \frac{\cancel{x^4}\left(\frac{1}{x^3}+\frac{2}{x}-24\right)}{\cancel{x^4}\left(\frac{1}{x^4}+3+\frac{1}{x}\right)} =$

$= \frac{0+0-24}{0+3+0} = -\frac{24}{3} = \underline{\underline{-8}}$

$\lim\limits_{x\to \frac{3\pi}{2}_-} \operatorname{arctan}\left(\frac{1}{\cos x}\right) = \operatorname{arctan}\left(\frac{1}{0_+}\right) = \underline{\underline{\frac{\pi}{2}}}$