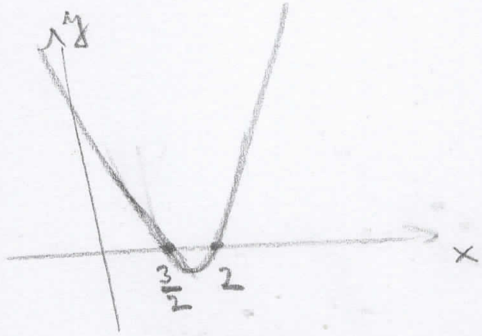
1. $f(x) = \dfrac{\sqrt{2x^2-7x+6}}{3x-1}$



- $2x^2 - 7x + 6 \geq 0$

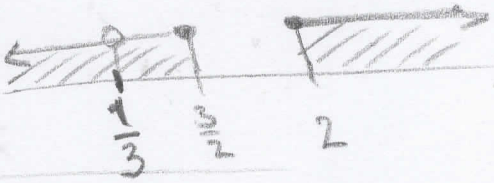
$D = 49 - 4\cdot 2\cdot 6 = 1$

$x_{1,2} = \dfrac{7 \pm 1}{4} = \begin{cases} 2 \\ \frac{3}{2} \end{cases}$

$(x-2)(x-\frac{3}{2}) \geq 0 \iff x \in (-\infty, \frac{3}{2}\rangle \cup \langle 2, \infty)$

- $3x - 1 \neq 0$

$x \neq \frac{1}{3}$



$D = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \frac{3}{2}\rangle \cup \langle 2, \infty)$

$\lim\limits_{x \to \frac{1}{3}^+} f(x) = \dfrac{\sqrt{\frac{2}{9} - \frac{7}{3} + 6}}{0_+} = +\infty$

$\lim\limits_{x \to \frac{1}{3}^-} f(x) = \dfrac{\sqrt{\frac{2}{9} - \frac{7}{3} + 6}}{0_-} = -\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to \infty} \dfrac{\cancel{|x|}\cdot\left(\sqrt{2 - \frac{7}{x} + \frac{6}{x^2}}\right)}{\cancel{x}\cdot\left(3 - \frac{1}{x}\right)} = \dfrac{\sqrt{2-0-0}}{3-0} = \dfrac{\sqrt{2}}{3}$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{(-1)\cancel{|x|}\cdot\sqrt{2 - \frac{3}{x} + \frac{6}{x^2}}}{\cancel{x}\left(3 - \frac{1}{x}\right)} = -\dfrac{\sqrt{2}}{3}$

(2.) $\lim_{x \to +\infty} x^2 \left(e^{\frac{1}{x^2}} - 1\right) = \lim_{x \to +\infty} \frac{e^{\frac{1}{x^2}} - 1}{\frac{1}{x^2}} =$

$\overset{L.P.}{=}_{\text{„}\frac{0}{0}\text{"}} \lim_{x \to \infty} \frac{\cancel{-\frac{2}{x^3}} \cdot e^{\frac{1}{x^2}}}{\cancel{-\frac{2}{x^3}}} = \lim_{x \to +\infty} e^{\frac{1}{x^2}} = e^0 = \underline{\underline{1}}$

(3.) $f(x) = 8 + 10x - 3x^2$, $k = -8$

x	0	3	4	$-\frac{2}{3}$
$f(x)$	8	11	0	0

$k = f'(x_0) = -8$

$f'(x) = 10 - 6x = -8$

$-6x = -18$

$\underline{\underline{x = 3}}$

$f(3) = 8 + 30 - 27 = 11$

tečný bod = $[3;11]$

tečna = $t: y = -8x + c$

$11 = -8 \cdot 3 + c$ $\quad c = 35$

$\boxed{t: y = -8x + 35}$

vrchol: $x = \frac{5}{3}$

$f\left(\frac{5}{3}\right) = 8 + \frac{50}{3} - \frac{25}{3} = \frac{49}{3}$

průsečíky s osami:

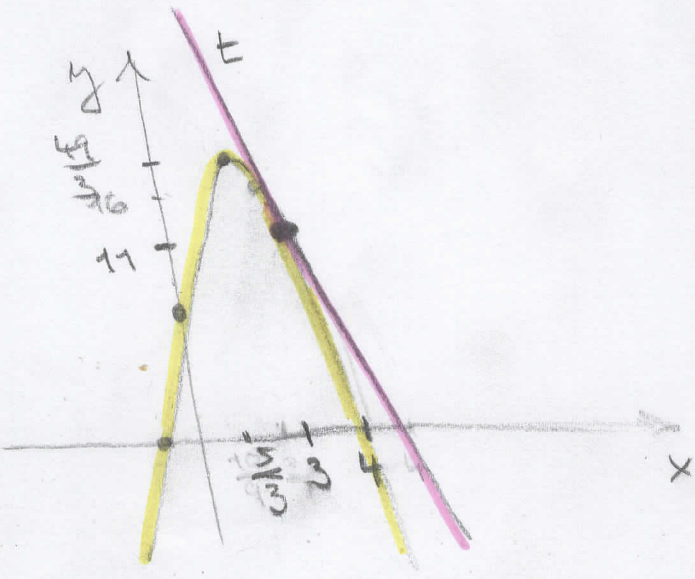
$f(0) = 8 \quad P_y = [0;8]$

$8 + 10x - 3x^2 = 0$

$3x^2 - 10x - 8 = 0$

$D = 100 + 96 = 196 = 14^2$

$x_{1,2} = \frac{10 \pm 14}{6} = \begin{cases} 4 \\ -\frac{2}{3} \end{cases}$



④ $f(x) = x \cdot e^{-\frac{x^2}{6}}$ $\qquad D = \mathbb{R}$

$$f'(x) = e^{-\frac{x^2}{6}} + x \cdot \left(-\frac{x}{3}\right) \cdot e^{-\frac{x^2}{6}} = e^{-\frac{x^2}{6}} \cdot \left(1 - \frac{x^2}{3}\right)$$

$$f'(x) = 0 \iff 1 - \frac{x^2}{3} = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$f(\pm\sqrt{3}) = \pm\sqrt{3}\, e^{-\frac{1}{2}}$$

$\searrow \quad \nearrow \quad \searrow$

$\underline{f'<0}$ $\quad -\sqrt{3} \quad$ $\underline{f'>0}$ $\quad \sqrt{3} \quad$ $\underline{f'<0}$

lok. maximum: $\left[\sqrt{3}; \sqrt{\frac{3}{e}}\right]$

lok. minimum: $\left[-\sqrt{3}; -\sqrt{\frac{3}{e}}\right]$

$$f''(x) = \left(-\frac{x}{3}\right) e^{-\frac{x^2}{6}} \cdot \left(1 - \frac{x^2}{3}\right) + e^{-\frac{x^2}{6}} \cdot \left(-\frac{2x}{3}\right) = e^{-\frac{x^2}{6}} \cdot \left(-\frac{x}{3} + \frac{x^3}{9} - \frac{2x}{3}\right)$$

$$= \frac{1}{9} e^{-\frac{x^2}{6}} \cdot \left(x^3 - 9x\right)$$

$$f''(x) = 0 \iff x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x = 0 \lor x = \pm 3$$

$\cap \quad \cup \quad \cap \quad \cup$

$\underline{f''<0}$ $\quad -3 \quad$ $\underline{f''>0}$ $\quad 0 \quad$ $\underline{f''<0}$ $\quad 3 \quad$ $\underline{f''>0}$

$\forall x \in (-\infty, -3) \cup (0, 3)$: $f''(x) < 0 \Rightarrow f$ je <u>konkávní</u> na $(-\infty, -3)$ a $(0, 3)$

$\forall x \in (-3, 0) \cup (3, \infty)$: $f''(x) > 0 \Rightarrow f$ je <u>konvexní</u> na $(-3, 0)$ a $(3, \infty)$

inflexní body: $[0; 0]$

$\left[3; 3e^{-\frac{3}{2}}\right]$

$\left[-3; -3e^{-\frac{3}{2}}\right]$

Funkce je lichá, neboť $\forall x \in D_f : f(-x) = -f(x)$

$$f(-x) = -x \cdot e^{-\frac{(-x)^2}{6}} = -xe^{-\frac{x^2}{6}} = -f(x)$$

průsečíky s osami: $f(0) = 0$

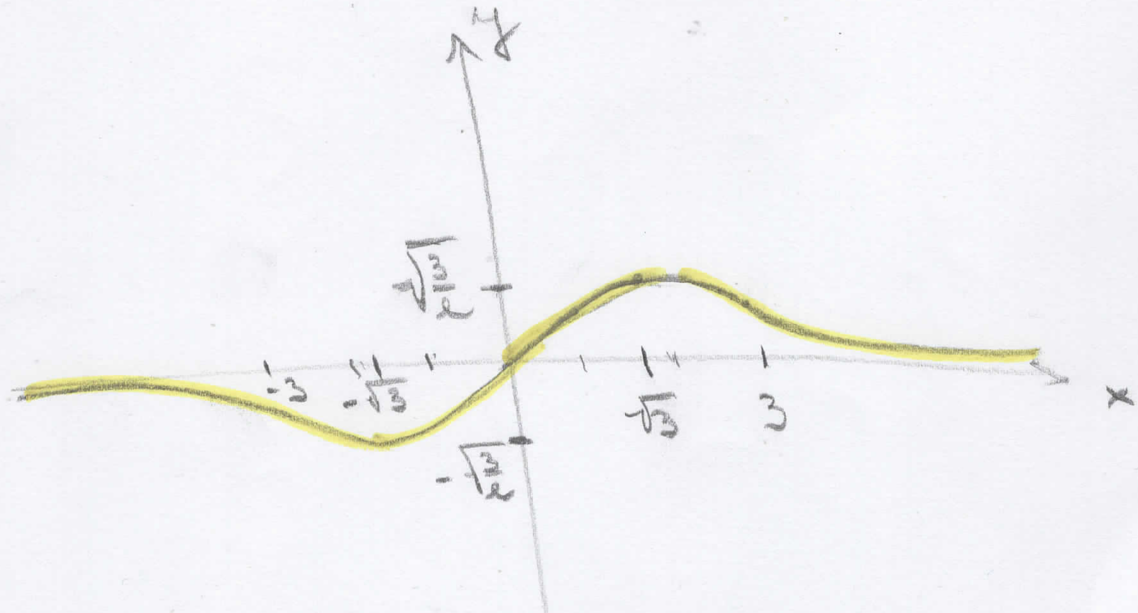
$$f(x) = 0 \iff xe^{-\frac{x^2}{6}} = 0 \iff x = 0$$

$$\lim_{x \to +\infty} xe^{-\frac{x^2}{6}} = \lim_{x \to \infty} \frac{x}{e^{\frac{x^2}{6}}} \overset{\text{L.P.}}{\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=}} \lim_{x \to \infty} \frac{1}{\frac{x}{3} \cdot e^{\frac{x^2}{6}}} = \frac{1}{\infty} = \underline{\underline{0}}$$

$$\lim_{x \to -\infty} xe^{-\frac{x^2}{6}} = \underline{\underline{0}}$$

$\Rightarrow$ vodorovná asymptota $y = 0$

x	0	$\sqrt{3}$ $\doteq 1{,}73$	$-\sqrt{3}$ $\doteq -1{,}73$	3	-3
$f(x)$	0	$\sqrt{\frac{3}{e}}$ $\doteq 1{,}05$	$-\sqrt{\frac{3}{e}}$ $\doteq -1{,}05$	$3e^{-\frac{3}{2}}$ $\doteq 0{,}67$	$-3e^{-\frac{3}{2}}$ $\doteq -0{,}67$

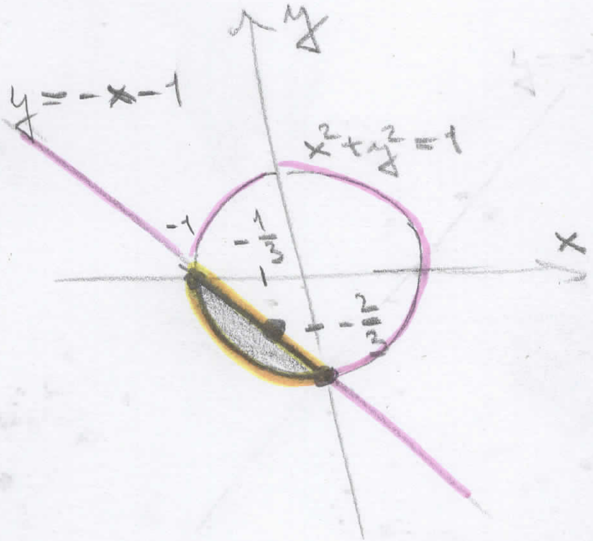


Obor hodnot: $H = \left\langle -\sqrt{\frac{3}{e}} ; \sqrt{\frac{3}{e}} \right\rangle$

(5.) $f(x,y) = 2x^2 + y^2$

$M: \quad x^2 + y^2 \leq 1$

$\quad\quad x + y + 1 \leq 0$



průsečíky přímky a kružnice:

$y = -x - 1$

$x^2 + y^2 = 1$

$x^2 + (-x-1)^2 = 1$

$x^2 + x^2 + 2x + 1 = 1 \quad |-1$

$\quad |:2$

$2x^2 + 2x = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$\underline{x = 0} \lor \underline{x = -1}$

Volné extrémy: $\frac{\partial f}{\partial x} = 4x = 0 \iff x = 0 \qquad [0,0] \notin M$

$\frac{\partial f}{\partial y} = 2y = 0 \iff y = 0$

Vázané extrémy: 1) $\underline{y = -x-1, \; x \in \langle -1, 0 \rangle}$

$g(x) := f(x, -x-1) = 2x^2 + (-x-1)^2 =$

$= 2x^2 + x^2 + 2x + 1 = 3x^2 + 2x + 1$

$g'(x) = 6x + 2 = 0 \iff \boxed{x = -\frac{1}{3}}$

$\boxed{y = -\frac{2}{3}}$

2) oblouk kružnice $\underbrace{x^2 + y^2 - 1}_{=: g(x,y)} = 0$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} 4x & 2y \\ 2x & 2y \end{vmatrix} = 8xy - 4xy = 4xy = 0$$

$\iff \underline{x = 0} \lor y = 0$

$y = -1 \quad\quad x = -1$

kandidáti : $[-1;0]$ $\quad f(-1,0) = 2$

$[0;-1]$ $\quad f(0,-1) = 1$

$[-\frac{1}{3};-\frac{2}{3}]$ $\quad f(-\frac{1}{3},-\frac{2}{3}) = \frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$

vázané minimum : $[-\frac{1}{3};-\frac{2}{3}]$, maximum : $[-1;0]$

6. $f(x,y,z) = x^2 + 4y^2 - 9z^2 + 6z$

$M: x^2 + y^2 + (3z-1)^2 \leq 9$

---

1) M° : $\frac{\partial f}{\partial x} = 2x = 0$

$\frac{\partial f}{\partial y} = 8y = 0$

$\frac{\partial f}{\partial z} = -18z + 6 = 0 \iff z = \frac{1}{3}$

$[0;0;\frac{1}{3}] \in M$ neboť $0^2 + 0^2 + (3\cdot\frac{1}{3}-1)^2 \leq 9$

2) ∂M : $L(x,y,z) = f(x,y,z) + \lambda \cdot g(x,y,z)$

$= x^2 + 4y^2 - 9z^2 + 6z + \lambda\cdot\left(x^2 + y^2 + (3z-1)^2 - 9\right)$

$\frac{\partial L}{\partial x} = 2x + 2\lambda x = 0 \iff 2x(1+\lambda) = 0$

$\frac{\partial L}{\partial y} = 8y + 2\lambda y = 0 \iff 2y(4+\lambda) = 0$

$\frac{\partial L}{\partial z} = -18z + 6 + 6\lambda\cdot(3z-1) = 0 \quad |:6$

$-3z + 1 + \lambda(3z-1) = 0 \implies \lambda = -1, z \in R$

$\vee \ \lambda \in R, z = \frac{1}{3}$

I. $\underline{\lambda = -1}$: $\boxed{y=0}$

$$-18z+6 \ -6(3z-1) = 0$$

$$-18z+6 - 18z+6 = 0$$

$$-36z+12 = 0$$

$$\boxed{z = \frac{1}{3}}$$

dosazením $y=0$, $z=\frac{1}{3}$ do $g(x,y,z)=0$:

$$x^2 + 0^2 + \left(3\cdot\frac{1}{3} - 1\right)^2 = 9$$

$$\boxed{x = \pm 3}$$

II. $\underline{\lambda = -4}$: $\boxed{x=0}$:

$$-18z+6+6\cdot 4\cdot(3z-1) = 0 \quad |:6$$

$$-3z+1+12z-4 = 0$$

$$9z-3 = 0$$

$$\boxed{z = \frac{1}{3}} \qquad \boxed{y = \pm 3}$$

III. $\underline{x=0,\ y=0}$: $(3z-1)^2 = 9$

$$(3z-1-3)(3z-1+3) = 0$$

$$(3z-4)(3z+2) = 0$$

$$\boxed{z = \frac{4}{3}} \vee \boxed{z = -\frac{2}{3}}$$

$f(0,0,\frac{1}{3}) = -1+2 = 1$

$f(\pm 3,0,\frac{1}{3}) = 9-1+2 = 10$

$f(0,\pm 3,\frac{1}{3}) = 36-1+2 = 37$ - maximum

$\left.\begin{array}{l} f(0,0,\frac{4}{3}) = -16+8 = -8 \\ f(0,0,-\frac{2}{3}) = -4-4 = -8 \end{array}\right\}$ minima