1. $f(x) = \dfrac{\sqrt{4x^2+x+1}}{2x+3}$

$$\boxed{D_f = \mathbb{R} \setminus \left\{-\frac{3}{2}\right\}}$$

$\forall x \in \mathbb{R}: 4x^2+x+1 \neq 0$

$D = b^2 - 4ac = 1^2 - 4 \cdot 4 \cdot 1 < 0$

$$\lim_{x \to -\frac{3}{2}^+} f(x) = \frac{\sqrt{9-\frac{3}{2}+1}}{0_+} = +\infty$$

$$\lim_{x \to -\frac{3}{2}^-} f(x) = \frac{\sqrt{9-\frac{3}{2}+1}}{0_-} = -\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\cancel{|x|} \cdot \sqrt{4+\frac{1}{x}+\frac{1}{x^2}}}{\cancel{x} \cdot \left(2+\frac{3}{x}\right)} = \frac{\sqrt{4+0+0}}{2+0} = \frac{2}{2} = 1$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\cancel{|x|} \cdot \sqrt{4+\frac{1}{x}+\frac{1}{x^2}}}{\underset{(-1)}{x} \cdot \left(2+\frac{3}{x}\right)} = -1$$

2. $$\lim_{n \to \infty} \frac{(n+2)^3 - (n-2)^3}{(2n-1)^{\frac{3}{2}} \cdot \sqrt{2n+1}} = \lim_{n \to \infty} \frac{n^3+6n^2+12n+8-(n^3-6n^2+12n-8)}{n^2 \cdot \left(2-\frac{1}{n}\right)^{\frac{3}{2}} \cdot \sqrt{2+\frac{1}{n}}}$$

$$= \lim_{n \to \infty} \frac{12n^2+16}{n^2 \cdot \left(2-\frac{1}{n}\right)^{\frac{3}{2}} \cdot \left(2+\frac{1}{n}\right)^{\frac{1}{2}}}$$

$$= \lim_{n \to \infty} \frac{\cancel{n^2}\left(12+\frac{16}{n^2}\right)}{\cancel{n^2}\left(2-\frac{1}{n}\right)^{\frac{3}{2}}\left(2+\frac{1}{n}\right)^{\frac{1}{2}}} = \frac{12+0}{(2-0)^{\frac{3}{2}}(2+0)^{\frac{1}{2}}}$$

$$= \frac{12}{2^{\frac{3}{2}+\frac{1}{2}}} = \frac{12}{2^2} = \frac{12}{4} = 3$$

3.

$f(x) = -12 + 20x - 3x^2 = \quad f(0) = -12$

$f'(x) = 20 - 6x$

| $x$ | $0$ | $6$ | $\frac{2}{3}$ |
|---|---|---|---|
| $f(x)$ | $-12$ | $0$ | $0$ |

$f'(0) = 20 = k$

tečna $t: y = 20x + c$

$[0; -12] \in t: \quad -12 = 20 \cdot 0 + c \implies c = -12$

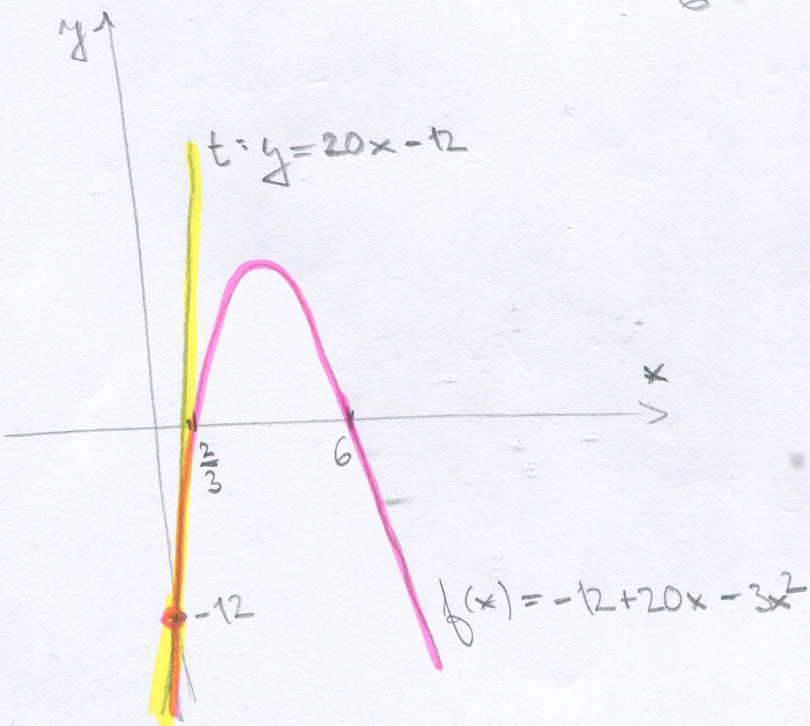
$$\boxed{t: y = 20x - 12}$$

$f(x) = 0 \iff -12 + 20x - 3x^2 = 0$

$3x^2 - 20x + 12 = 0$

$D = 400 - 4 \cdot 3 \cdot 12 = 400 - 144 = 256$

$x_{1,2} = \frac{20 \pm 16}{6} = \left\langle \begin{array}{l} 6 \\ \frac{2}{3} \end{array} \right.$

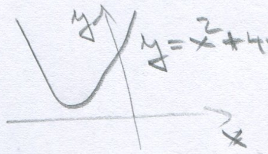


4. $f(x) = \ln(x^2+4x+5)$

| $x$ | $-2$ | $-1$ | $-3$ | $0$ |
|---|---|---|---|---|
| $f(x)$ | $0$ | $\ln 2$ | $\ln 2$ | $\ln 5$ |

$\forall x \in \mathbb{R}: x^2+4x+5 > 0$

$D = 4^2 - 4\cdot1\cdot5 < 0$




$\boxed{D_f = \mathbb{R}}$

$$f'(x) = \frac{2x+4}{x^2+4x+5}$$

$f'(x) = 0 \iff x = -2$

$f' < 0$ ↘ $-2$ ↗ $f' > 0$

$$f''(x) = \frac{2(x^2+4x+5) - (2x+4)^2}{(x^2+4x+5)^2} = \frac{2x^2+8x+10-4x^2-16x-16}{(x^2+4x+5)^2}$$

$$= \frac{-2x^2-8x-6}{(x^2+4x+5)^2} = (-2)\,\frac{x^2+4x+3}{(x^2+4x+5)^2}$$

$f''(x) = 0 \iff x^2+4x+3 = 0$

$(x+3)(x+1) = 0$

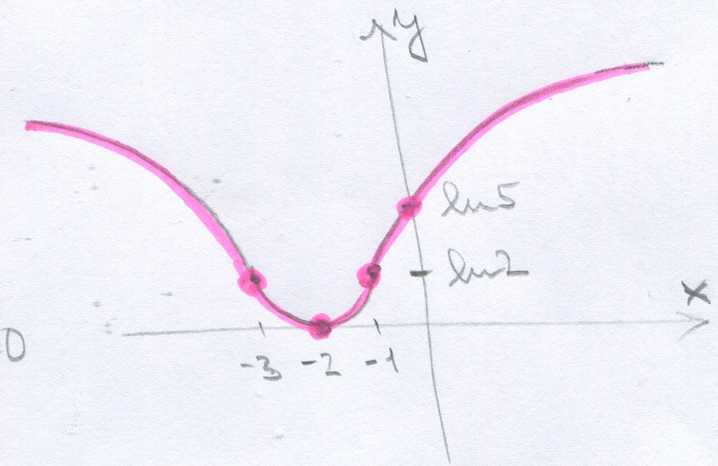
$x = -3 \lor x = -1$

$f'' < 0$ $-3$ $f'' > 0$ $-1$ $f'' < 0$

$$\lim_{x\to\pm\infty} \ln(x^2+4x+5) = \lim_{x\to\pm\infty} \ln\left(x^2\left(1+\frac{4}{x}+\frac{5}{x^2}\right)\right) = \ln(\infty) = \underline{\underline{+\infty}}$$

$f(-x) = \ln(x^2-4x+5)$

$\left.\begin{array}{l} f(-x) \neq f(x) \\ f(-x) \neq -f(x) \end{array}\right\}$ není sudá ani lichá




$f(x) = 0 \iff \ln(x^2+4x+5) = 0$

$x^2+4x+5 = 1$

$x^2+4x+4 = 0$

$(x+2)^2 = 0$

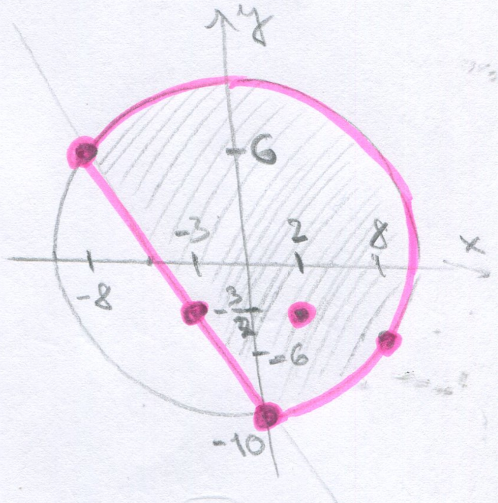
$\underline{x = -2}$ $P_x = [-2;0]$

$H_f = \langle 0, \infty)$

5. $f(x,y) = x^2 + y^2 - 4x + 3y$

$M: \; x^2 + y^2 \le 100$

$\quad y \ge -2x - 10$




průsečíky přímky a kružnice:

$y = -2x - 10$

$x^2 + y^2 = 100$

$x^2 + (-2x-10)^2 = 100$

$x^2 + 4x^2 + 40x + 100 = 100$

$5x^2 + 40x = 0$

$5x(x+8) = 0$

$\underline{x = 0} \; \vee \; \underline{x = -8}$

$y = -10 \qquad y = 6$

1) $M^\circ$: $\frac{\partial f}{\partial x} = 2x - 4 = 0 \iff x = 2$

$\frac{\partial f}{\partial y} = 2y + 3 = 0 \iff y = -\frac{3}{2}$

2) $\partial M$: 1. úsečka $y = -2x - 10$, $x \in \langle -8, 0 \rangle$

$g(x) := f(x, -2x-10) = x^2 + (-2x-10)^2 - 4x + 3(-2x-10$

$= 5x^2 + 40x + 100 - 10x - 30$

$= 5x^2 + 30x + 70$

$g'(x) = 10x + 30 = 0 \iff x = -3$

$y = -4$

II. oblouk kružnice $x^2+y^2=100$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x-4 & 2y+3 \\ 2x & 2y \end{vmatrix} =$$

$$= (2x-4)2y - 2x(2y+3)$$

$$= 4xy - 8y - 4xy - 6x = 0$$

$$-6x = 8y \quad |:8$$

$$\boxed{y = -\frac{3}{4}x}$$

$$x^2+y^2=100$$

$$x^2+\frac{9}{16}x^2=100$$

$$\frac{25}{16}x^2=100 \quad |:25 \quad |\cdot 16$$

$$x^2=64$$

$$x=\pm 8$$

$$y=\mp 6$$

kandidáti: $[0;-10]$ $f(0,-10)=70$

maximum $[-8;6]$ $f(-8,6)=64+36+32+18=\underline{150}$

$[-3;-4]$ $f(-3,-4)=9+16+12-12=25$

minimum $[2;-\frac{3}{2}]$ $f(2,-\frac{3}{2})=4+\frac{9}{4}-8-\frac{9}{2}=-\frac{25}{4}$

$[8;-6]$ $f(8,-6)=64+36-32-18=50$

(6.) $f(x,y,z) = xz - y$

$M: \underbrace{x^2+y^2+z^2-9}_{=:g_1(x,y,z)} = 0 \quad ; \quad \underbrace{x+yz}_{=:g_2(x,y,z)} = 0 .$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial z} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial z} \end{vmatrix} = \begin{vmatrix} z & -1 & x \\ 2x & 2y & 2z \\ 1 & z & y \end{vmatrix} =$$

$$= 2y^2 z + 2x^2 z - 2z - (\cancel{2yx} + 2z^3 \cancel{-2xy})$$

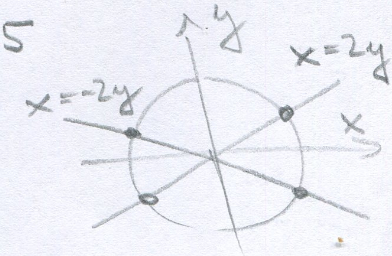
$$= 2z(y^2 + x^2 - z^2 - 1) = 0$$

I. $z = 0$
$x = 0$
$y = \pm 3$

II. $y^2 + x^2 - z^2 - 1 = 0$
$y^2 + x^2 + z^2 - 9 = 0$ $+$

$2y^2 + 2x^2 - 10 = 0$

$x^2 + y^2 = 5$




$z = \pm 2$

$x = \mp 2y$

$4y^2 + y^2 = 5$

$5y^2 = 5 \Rightarrow y = \pm 1$

kandidáti: $[0; \pm 3; 0]$, $[\mp 2; \pm 1; 2]$, $[\pm 2; \pm 1; -2]$

$f(0,3,0) = -3$

$f(0,-3,0) = 3$

$f(-2,1,2) = -5$ min

$f(2,-1,2) = 5$ max

$f(2,1,-2) = -5$ min

$f(-2,-1,-2) = 5$ max