1. $f(x) = \dfrac{\sqrt{3-x}}{x^2-16}$ $\qquad D = (-\infty, 3\rangle \setminus \{-4\}$

$x^2-16 \neq 0 \quad \wedge \quad 3-x \geq 0$

$(x-4)(x+4) \neq 0 \qquad x \leq 3$

$x \neq \pm 4$

$$\lim_{x \to 3_-} f(x) = f(3) = \frac{\sqrt{0}}{9-16} = \underline{\underline{0}}$$

$$\lim_{x \to -4_+} f(x) = \frac{\sqrt{7}}{0_-} = \underline{\underline{-\infty}}$$

$$\lim_{x \to -4_-} f(x) = \frac{\sqrt{7}}{0_+} = \underline{\underline{+\infty}}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} \frac{x^{\frac{1}{2}} \cdot \sqrt{\frac{3}{x}+1}}{x^2 \cdot \left(1 - \frac{16}{x^2}\right)} = \underbrace{\lim_{x \to \infty} \frac{1}{x^{\frac{3}{2}}}}_{0} \cdot \underbrace{\lim_{x \to \infty} \frac{\sqrt{\frac{3}{x}+1}}{1-\frac{16}{x^2}}}_{1} = \underline{\underline{0}}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(-x)$$

2. $$\lim_{n \to \infty} \frac{5 \cdot \left(\frac{1}{16}\right)^{\frac{n}{2}} + 2\left(\frac{2\sqrt{2}}{5}\right)^{2n}}{3 \cdot \left(\frac{2}{7}\right)^n + 25 \cdot \left(\frac{8}{25}\right)^{n+1}} = \lim_{n \to \infty} \frac{5 \cdot \left(\frac{1}{4}\right)^n + 2 \cdot \left(\frac{8}{25}\right)^n}{3 \cdot \left(\frac{2}{7}\right)^n + 25 \cdot \left(\frac{8}{25}\right)^{n+1}}$$

$$= \lim_{n \to \infty} \frac{\cancel{\left(\frac{8}{25}\right)^n} \cdot \left(5 \cdot \left(\frac{1}{4}\right)^n \cdot \left(\frac{25}{8}\right)^n + 2\right)}{\cancel{\left(\frac{8}{25}\right)^n} \cdot \left(3 \cdot \left(\frac{2}{7}\right)^n \cdot \left(\frac{25}{8}\right)^n + \cancel{25} \cdot \frac{8}{\cancel{25}}\right)}$$

$$= \lim_{n \to \infty} \frac{5 \cdot \left(\frac{25}{32}\right)^n + 2}{3 \cdot \left(\frac{25}{28}\right)^n + 8} = \frac{5 \cdot 0 + 2}{3 \cdot 0 + 8} = \underline{\underline{\frac{1}{4}}}$$

$\frac{8}{25} > \frac{1}{4}$

$\frac{32}{100} > \frac{25}{100}$

$\frac{8}{25} > \frac{2}{7}$

$\frac{56}{175} > \frac{50}{175}$

$$\boxed{\lim_{n \to \infty} q^n = 0 \text{ pro } q \in \langle 0, 1)}$$

3.

$$f(x) = \frac{x+1}{x-1} = \frac{x-1+2}{x-1} = 1 + \frac{2}{x-1}$$

asymptoty : $x = 1$
$y = 1$

$$f(2) = \frac{2+1}{2-1} = \frac{3}{1} = 3$$ tečný bod: $[2;3]$

$$f'(x) = \frac{x-1-(x+1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$$ $f'(2) = -2 = k$

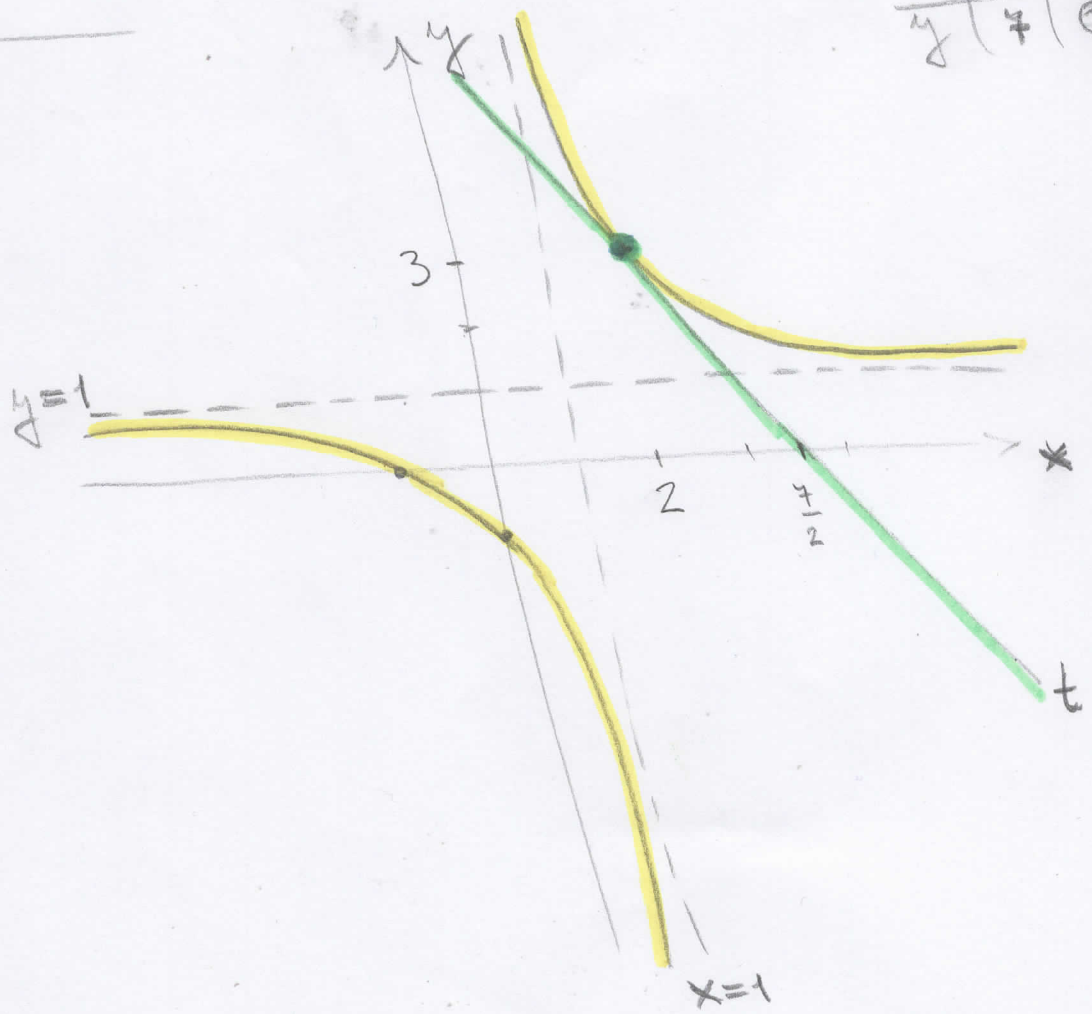
tečna t: $y = -2x + c$

$[2;3] \in t$: $3 = -2 \cdot 2 + c \implies c = 7$

$\boxed{t: y = -2x + 7}$

x	0	$\frac{7}{2}$
y	7	0

x	0	-1
$f(x)$	-1	0



(4) $f(x) = e^{3-x^2}$ $\qquad D_f = \mathbb{R}$

x	0	$-$	$\pm\frac{\sqrt{2}}{2}$
$f(x)$	$e^3 \doteq 20{,}1$	0	$e^{\frac{5}{2}} \doteq 12{,}2$

$f'(x) = (-2x)\, e^{3-x^2}$

$f'(x) = 0 \iff x = 0$

$\nearrow \quad \searrow$

$f' > 0 \quad 0 \quad f' < 0$

$f''(x) = (-2)e^{3-x^2} + (-2x)^2 \cdot e^{3-x^2} = e^{3-x^2} \cdot (4x^2 - 2)$

$f''(x) = 0 \iff 4x^2 - 2 = 0 \quad |:2$

$2x^2 - 1 = 0$

$x = \pm\frac{\sqrt{2}}{2}$

$\cup \quad \cap \quad \cup$

$f'' > 0 \quad -\frac{\sqrt{2}}{2} \quad f'' < 0 \quad \frac{\sqrt{2}}{2} \quad f'' > 0$

inflexní body: $[\pm\frac{\sqrt{2}}{2}; e^{\frac{5}{2}}]$

$\forall x \in \mathbb{R}: f(-x) = e^{3-(-x)^2} = e^{3-x^2} = f(x) \implies f$ je <u>sudá</u>

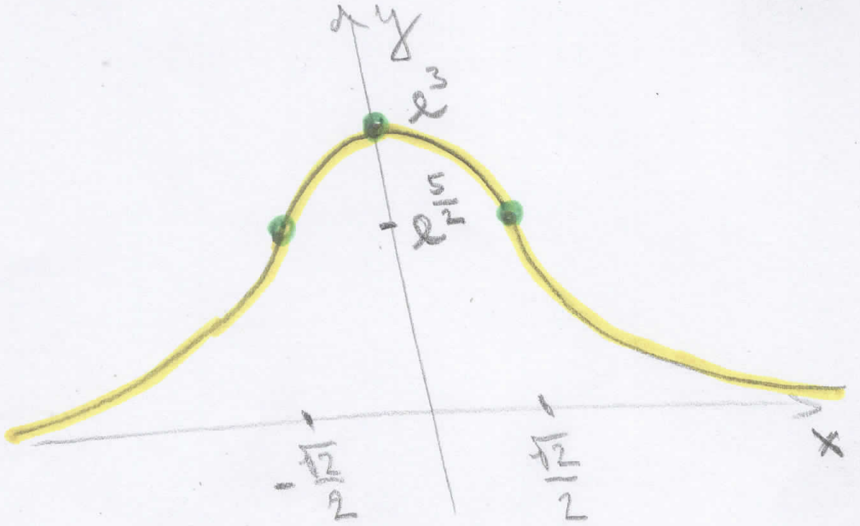
$\lim\limits_{x \to \pm\infty} f(x) = e^{-\infty} = \underline{\underline{0}} \implies y = 0$ je vodorovná asymptota

P_x nemá neboť $\forall x \in \mathbb{R}: e^{3-x^2} \neq 0$

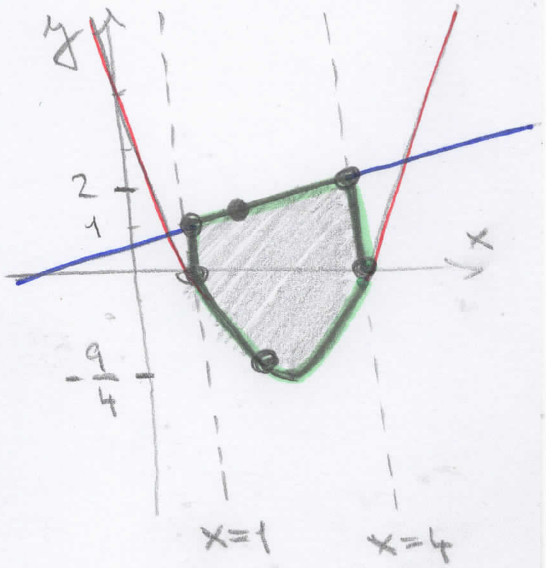
$P_y = [0; e^3]$

Obor hodnot

$H_f = (0, e^3\rangle$



5. $f(x,y) = (x-2)^2 + y$



M: $1 \le x \le 4$

$$x^2-5x+4 \le y \le \frac{x+2}{3}$$

$$h(x) := x^2-5x+4 = (x-1)(x-4)$$

$$h\left(\frac{5}{2}\right) = \frac{25}{4} - \frac{25}{2} + 4 = \frac{-25+16}{4} = -\frac{9}{4}$$

M°:

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 2(x-2) = 0 \iff x = 2 \\ \frac{\partial f}{\partial y} &= 1 \neq 0 \end{aligned}\right\} \text{nemá extrémy v } M^\circ$$

∂M: I. úsečka $y = \frac{x+2}{3}$, $x \in \langle 1,4 \rangle$

$$g(x) := f\left(x, \frac{x+2}{3}\right) = (x-2)^2 + \frac{x+2}{3}$$

$$g'(x) = 2(x-2) + \frac{1}{3} = 0 \quad | \cdot 3$$

$$6x - 12 + 1 = 0$$

$$x = \frac{11}{6} \qquad y = \frac{\frac{11}{6}+2}{3} = \frac{23}{18}$$

II. úsečka $x=1$, $y \in \langle 0,1 \rangle$: $g(y) := f(1,y) = 1+y$

$$g'(y) = 1 \neq 0 \quad \forall y \in \langle 0,1 \rangle$$

III. úsečka $x=4$, $y \in \langle 0,2 \rangle$: $g(y) := f(4,y) = 4+y$

$$g'(y) = 1 \neq 0 \quad \forall y \in \langle 0,1 \rangle$$

IV. parabola $y = x^2 - 5x + 4$

$$g(x) := f(x, x^2-5x+4) = (x-2)^2 + x^2 - 5x + 4 =$$

$$= x^2 - 4x + 4 + x^2 - 5x + 4 = 2x^2 - 9x + 8$$

$$g'(x) = 4x - 9 = 0 \iff x = \frac{9}{4}$$

$$y = \frac{81}{16} - 5 \cdot \frac{9}{4} + 4 = \frac{81 - 180 + 64}{16}$$

$$= -\frac{35}{16}$$

seznam kandidátů: $[1;0], [4;0], [4;2], [1;1]$

$$\left[\frac{11}{6}; \frac{23}{18}\right]; \left[\frac{9}{4}; -\frac{35}{16}\right]$$

$$f(1,0) = 1$$

$$f(4,0) = 4$$

$$f(4,2) = 6 \quad \text{maximum}$$

$$f(1,1) = 2$$

$$f\left(\frac{11}{6}, \frac{23}{18}\right) = \frac{1}{36} + \frac{23}{18} = \frac{47}{36}$$

$$f\left(\frac{9}{4}, -\frac{35}{16}\right) = \frac{1}{16} - \frac{35}{16} = -\frac{34}{16} = -\frac{17}{8} \quad \text{minimum}$$

(6.) $f(x,y,z) = xy + 4z^2$

$M: x^2 + y^2 + z^2 \leq 18$

$$\frac{\partial f}{\partial x} = y = 0$$
$$\frac{\partial f}{\partial y} = x = 0$$
$$\frac{\partial f}{\partial z} = 8z = 0$$

stacionární bod $[0,0,0] \in M^0$

$L(x,y,z) = f(x,y,z) + \lambda g(x,y,z) =$

$= xy + 4z^2 + \lambda(x^2 + y^2 + z^2 - 18)$

$$\frac{\partial L}{\partial x} = y + 2\lambda x = 0$$
$$\frac{\partial L}{\partial y} = x + 2\lambda y = 0$$
$$\frac{\partial L}{\partial z} = 8z + 2\lambda z = 2z(4+\lambda) = 0$$

$\Leftrightarrow \underline{\lambda = -4} \vee \underline{z = 0}$

I. $\underline{\lambda = -4}$: $y = 8x$

$x + 2\cdot(-4)\cdot 8x = 0 \Rightarrow x = 0$
$y = 0$
$z = \pm\sqrt{18}$

II. $\underline{\lambda = -\frac{y}{2x}}$: $x - \frac{y^2}{x} = 0 \quad |\cdot x$

$x^2 - y^2 = 0$

$x = \pm y$

$x = \pm y$

$z = 0: \quad 2y^2 = 18$

$y^2 = 9$

$y = \pm 3$

$[\pm 3; 3, 0]$

$[\pm 3; -3; 0]$

$4 - \frac{y}{2x} = 0$

$8x - y = 0$

$y = 8x \quad \dots \quad x = 0, y = 0, z = \pm\sqrt{18}$

III. $\lambda = -\frac{x}{2y}: \quad y - \frac{x^2}{y} = 0$

$y^2 - x^2 = 0$

$y = \pm x \quad \dots$

seznam kandidátů:

$[0,0,0]$	$f(0,0,0) = 0$
$[0,0,\sqrt{18}]$	$f(0,0,\sqrt{18}) = 72$
$[0,0,-\sqrt{18}]$	$f(0,0,-\sqrt{18}) = 72$
$[3,3,0]$	$f(3,3,0) = 9$
$[-3;3;0]$	$f(-3,3,0) = -9$
$[3;-3;0]$	$f(3,-3,0) = -9$
$[-3;-3;0]$	$f(-3,-3,0) = 9$

$\left.\begin{array}{l} f(0,0,\sqrt{18}) \\ f(0,0,-\sqrt{18}) \end{array}\right\}$ maxima

$\left.\begin{array}{l} f(-3,3,0) \\ f(3,-3,0) \end{array}\right\}$ minima