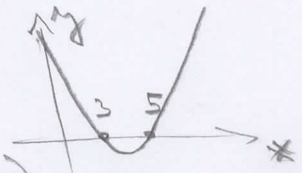
1.

$$f(x) = \frac{\sqrt{x^2 - 8x + 15}}{3x - 6}$$

- $x^2 - 8x + 15 \geq 0$

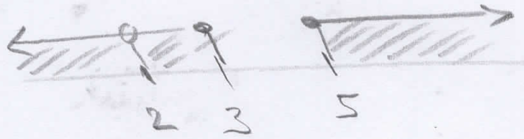
  $(x-3)(x-5) \geq 0$

  $x \in (-\infty, 3\rangle \cup \langle 5, \infty)$



- $3x - 6 \neq 0$

  $x \neq 2$



$$D = (-\infty, 2) \cup (2, 3\rangle \cup \langle 5, \infty)$$

$$\lim_{x \to +\infty} \frac{\sqrt{x^2 + 8x + 15}}{3x - 6} = \lim_{x \to +\infty} \frac{\cancel{|x|} \cdot \sqrt{1 - \frac{8}{x} + \frac{15}{x^2}}}{\cancel{x} \cdot \left(3 - \frac{6}{x}\right)} = \frac{\sqrt{1 - 0 + 0}}{3 - 0} = \frac{1}{3}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 - 8x + 15}}{3x - 6} = \lim_{x \to -\infty} \frac{\cancel{|x|} \sqrt{1 - \frac{8}{x} + \frac{15}{x^2}}}{\underbrace{\cancel{x}}_{(-1)} \left(3 - \frac{6}{x}\right)} = \frac{\sqrt{1 - 0 - 0}}{(-1)(3 - 0)} = -\frac{1}{3}$$

$$\lim_{x \to 2_+} f(x) = \frac{\sqrt{3}}{0_+} = +\infty$$

$$\lim_{x \to 2_-} f(x) = \frac{\sqrt{3}}{0_-} = -\infty$$

2.

$$\lim_{x \to 1} \frac{\ln(5x^2 - 4x)}{x^2 + 3x - 4} \underset{\text{" } \frac{0}{0} \text{ "}}{\overset{\text{L.P.}}{=}} \lim_{x \to 1} \frac{\frac{1}{5x^2 - 4x} \cdot (10x - 4)}{2x + 3} =$$

$$= \frac{\frac{1}{5 - 4} \cdot (10 - 4)}{2 + 3} = \frac{6}{5}$$

3.

$f(x) = x^2 + 12x + 35 = (x+6)^2 - 1$ Vrchol: $[-6; -1]$

$f'(x) = 2x + 12 \overset{?}{=} -4$

$2x = -16$

$x = -8$

$f(-8) = 64 + 12\cdot(-8) + 35 = 99 - 96 = 3$

tečný bod: $[-8; 3]$

průsečíky paraboly s osami: $\underline{x=0}$: $f(0) = 35$

$y=0$: $x^2 + 12x + 35 = 0$

$(x+5)(x+7) = 0$

$\underline{x=-5} \vee \underline{x=-7}$

$P_{x_1} = [-5; 0]$

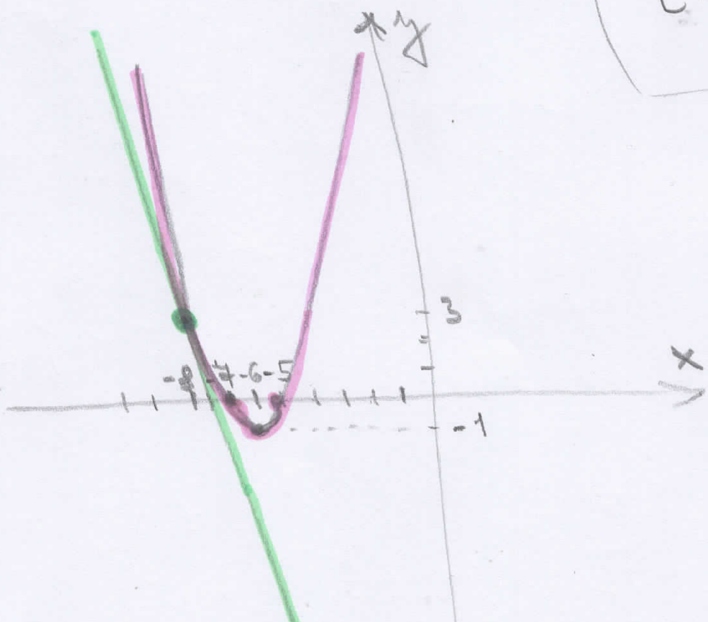
$P_{x_2} = [-7; 0]$

$P_y = [0; 35]$

tečna: $t: y = -4x + q$

$[-8; 3] \in t: 3 = (-4)(-8) + q \Rightarrow q = -29$

$t: y = -4x - 29$



4.

$f(x) = x - 4\sqrt{x}$ $\qquad$ $D_f = \langle 0, \infty)$

$f'(x) = 1 - 4 \cdot \frac{1}{2\sqrt{x}} = 1 - \frac{2}{\sqrt{x}}$

x	4	0	16
$f(x)$	-4	0	0

$f'(x) = 0 \iff 1 - \frac{2}{\sqrt{x}} = 0$

$1 = \frac{2}{\sqrt{x}}$

$\sqrt{x} = 2$

$x = 4$

$f(4) = 4 - 4\cdot\sqrt{4} = -4$

$\searrow \qquad \nearrow$

$0 \quad f' < 0 \quad 4 \quad f' > 0$

lokální minimum: $[4; -4]$

$f''(x) = (-2)\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\sqrt{x^3}} = \frac{1}{\sqrt{x^3}}$

$\forall x \in D_f: f''(x) > 0 \implies$ Funkce f je ryze konvexní

průsečíky s osami: $f(0) = 0 \quad P_y = [0, 0]$

$y = 0 \iff x - 4\sqrt{x} = 0$

$x = 4\sqrt{x} \quad |^2$

$x^2 = 16x$

$x^2 - 16x = 0$

$x(x - 16) = 0$

$\underline{x = 0} \lor \underline{x = 16}$

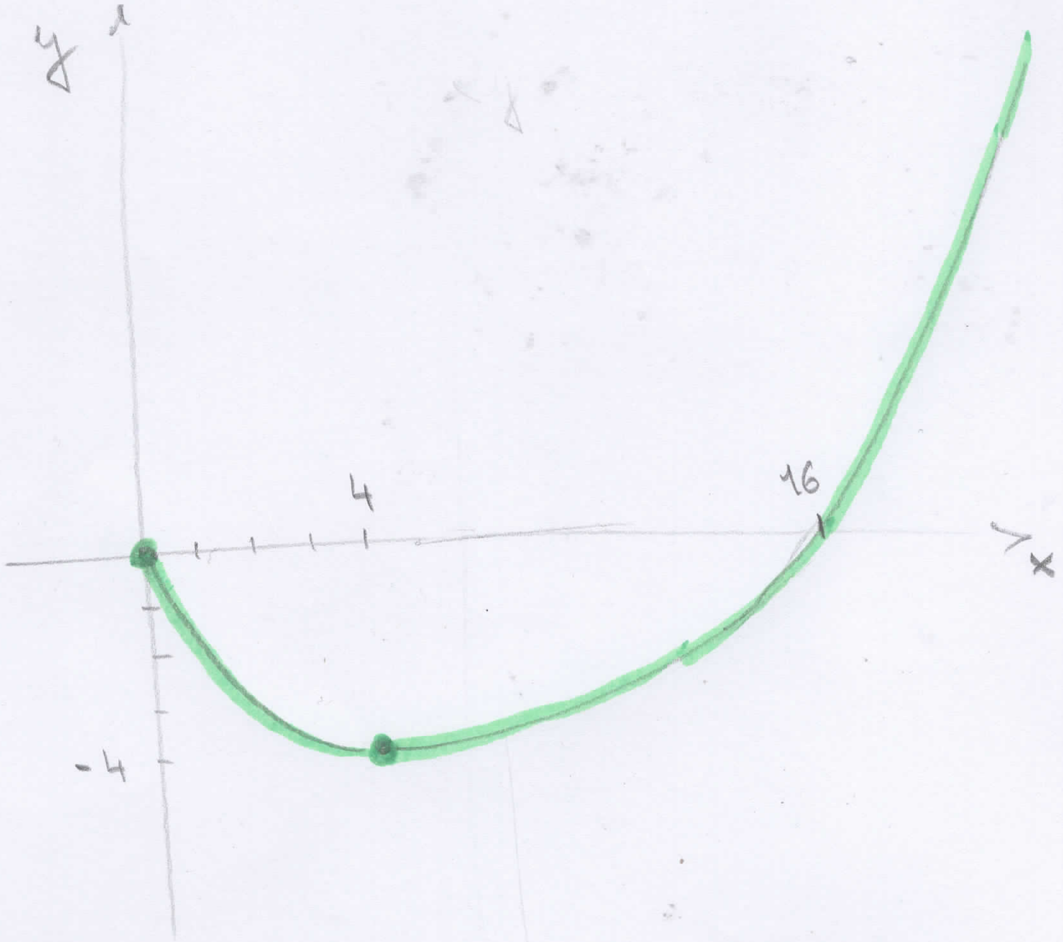
šikmá asymptota: $y = kx + q$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x - 4\sqrt{x}}{x} = \lim_{x\to\infty}\left(1 - \frac{4}{\sqrt{x}}\right) = 1$

$q = \lim_{x\to\infty}\left(f(x) - k\cdot x\right) = \lim_{x\to\infty}\left(x - 4\sqrt{x} - 1\cdot x\right) = -\infty$

$\implies$ nemá šikmou asymptotu

Obor hodnot : $H = \langle -4, \infty)$

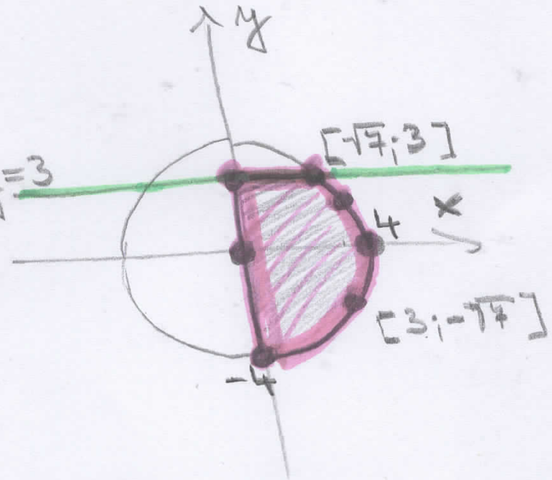


5. $f(x,y) = y^2 - x^2 + 12x$

$M:\ x^2 + y^2 \le 16,\ x \ge 0,\ y \le 3$

průsečíky přímky $y=3$ a kružnice:

$$x^2 + y^2 = 16$$
$$y = 3$$
$$x^2 + 9 = 16$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$



Vnitřek M: $\frac{\partial f}{\partial x} = -2x + 12 = 0 \iff x = 6$, $\frac{\partial f}{\partial y} = 2y = 0 \iff y = 0$ $\Big\}\ [6;0] \notin M$

Hranice M: I. úsečka $x = 0,\ y \in \langle -4, 3\rangle$

$g(y) := f(0,y) = y^2$

$g'(y) = 2y = 0 \iff \boxed{y=0}$ $\quad g'<0 \quad 0 \quad g'>0$

II. úsečka $y=3,\ x \in \langle 0, \sqrt{7}\rangle$: $g(x) := f(x,3) = 9 - x^2 + 12x$

$g'(x) = -2x + 12 = 0 \iff x = 6 \notin \langle 0, \sqrt{7}\rangle$

III. oblouk $x^2 + y^2 = 16$: $\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} -2x+12 & 2y \\ 2x & 2y \end{vmatrix} =$

$0 = g(x,y) := x^2 + y^2 - 16$

$= (-2x+12)2y - 4xy = 2y(-2x+12-2x) = 2y(12-4x) = 0$

$\underline{y=0} \lor \underline{x=3}$

$\underline{y=0}$: $x = \pm 4$

$\underline{x=3}$: $y = \pm\sqrt{7}$

seznam kandidátů:

$[0;0]$	$f(0,0) = 0$	minimum
$[0;3]$	$f(0,3) = 9$	
$[0;-4]$	$f(0,-4) = 16$	
$[4;0]$	$f(4,0) = -16 + 48 = 32$	
$[\sqrt{7};3]$	$f(\sqrt{7},3) = 9 - 7 + 12\sqrt{7} \doteq 33{,}75$	
$[3;-\sqrt{7}]$	$f(3;-\sqrt{7}) = 7 - 9 + 36 = 34$	maxima
$[3;\sqrt{7}]$	$f(3;\sqrt{7}) = 34$	maxima

(6.) $f(x,y,z) = 4xy - z^2$

$g_1(x,y,z) = 4x^2 + 4y^2 - z - 9 = 0$

$g_2(x,y,z) = z^2 - 1 = 0$

rozvoj podle 3. řádku

Jakobián: $\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial z} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial z} \end{vmatrix} = \begin{vmatrix} 4y & 4x & -2z \\ 8x & 8y & -1 \\ 0 & 0 & 2z \end{vmatrix} = 2z \cdot (-1)^{3+3} \begin{vmatrix} 4y & 4x \\ 8x & 8y \end{vmatrix}$

$= 2z \cdot (32y^2 - 32x^2) = 64z(y-x)(y+x) = 0$

$\iff \underline{z=0} \vee \underline{y=x} \vee \underline{y=-x}$

$g_2(x,y,z) = 0 \iff z = \pm 1$

I. $g_1(x,-x,\pm 1) = 8x^2 \mp 1 - 9 = 0$ $<$ $8x^2 - 8 = 0 \iff \underline{x = \pm 1,\ y = \mp 1}$; $8x^2 - 10 = 0 \iff \underline{x = \pm\frac{\sqrt{5}}{2}\ \ y = \mp\frac{\sqrt{5}}{2}}$

II. $g_1(x,x,\pm 1) = 8x^2 \mp 1 - 9 = 0$ $<$ $8x^2 - 8 = 0 \iff x = \pm 1$; $8x^2 - 10 = 0 \iff x = \pm\frac{\sqrt{5}}{2}$

kandidáti: $[\pm 1, \mp 1, \pm 1]$, $[\pm\frac{\sqrt{5}}{2}; \mp\frac{\sqrt{5}}{2}; \mp 1]$, $[\pm 1; \pm 1; 1]$; $[\pm\frac{\sqrt{5}}{2}; \pm\frac{\sqrt{5}}{2}; -1]$

$f(1,-1,1) = -4-1 = -5$

$f(-1,1,1) = -5$

$\left.\begin{array}{l} f\left(\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}, -1\right) = -5-1 = \underline{\underline{-6}} \\ f\left(-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}, -1\right) = \underline{\underline{-6}} \end{array}\right\}$ minima

$f(1,1,1) = 3$

$f(-1,-1,1) = 3$

$\left.\begin{array}{l} f\left(\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}, -1\right) = \underline{\underline{4}} \\ f\left(-\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}, -1\right) = \underline{\underline{4}} \end{array}\right\}$ maxima