

$$1) \quad f(x) = \sin(2x) \quad |_{x=0} = 0$$

$$f'(x) = 2\cos(2x) \quad |_{x=0} = 2$$

$$f''(x) = -4\sin(2x) \quad |_{x=0} = 0$$

$$f'''(x) = -8\cos(2x) \quad |_{x=0} = -8$$

$$T_{f,0}^3(x) = \underbrace{f(0)}_0 + \underbrace{f'(0)}_2 \cdot x + \underbrace{\frac{f''(0)}{2!}}_0 \cdot x^2 + \underbrace{\frac{f'''(0)}{3!}}_{-\frac{8}{6}} \cdot x^3 = \underline{\underline{2x - \frac{4}{3}x^3}}$$

$$2) \quad f(x) = \frac{1}{x^2+1} \quad |_{x=0} = 1$$

$$f'(x) = -\frac{2x}{(x^2+1)^2} \quad |_{x=0} = 0$$

$$f''(x) = -\frac{2(x^2+1)^2 - 2x \cdot 2x \cdot 2(x^2+1)}{(x^2+1)^4} \quad |_{x=0} = -2$$

$$T_{f,0}^2(x) = \underbrace{f(0)}_1 + \underbrace{f'(0)}_0 \cdot x + \underbrace{\frac{f''(0)}{2!}}_{-\frac{2}{2}} \cdot x^2 = \underline{\underline{1 - x^2}}$$

$$3) \quad f_1(x) = \ln x \quad |_{x=1} = 0$$

$$f_1'(x) = \frac{1}{x} \quad |_{x=1} = 1$$

$$f_1''(x) = -\frac{1}{x^2} \quad |_{x=1} = -1$$

$$T_{\ln x, 1}^2(x) = 1 \cdot (x-1) - \frac{1}{2}(x-1)^2$$

$$\ln 0,9 \approx 0,9 - 1 - \frac{1}{2}(0,9-1)^2$$

$$= -0,1 - \frac{1}{2} \cdot \frac{1}{100} = \underline{\underline{-0,105}}$$

5000

$$f_2(x) = \sqrt{x} \quad |_{x=1} = 1$$

$$f_2'(x) = \frac{1}{2\sqrt{x}} \quad |_{x=1} = \frac{1}{2}$$

$$f_2''(x) = -\frac{1}{4x^{3/2}} \quad |_{x=1} = -\frac{1}{4}$$

$$T_{\sqrt{x}, 1}^2(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

$$\sqrt{1,04} \approx 1 + 0,02 - \frac{1}{8} \cdot 0,0016$$

$$= 1,0198$$

20000