$\Omega: \sqrt{x^2+y^2} \le z \le 2-x^2-y^2$

VÁLCOVÉ SOUŘADNICE:

$0 \le \varphi < 2\pi$

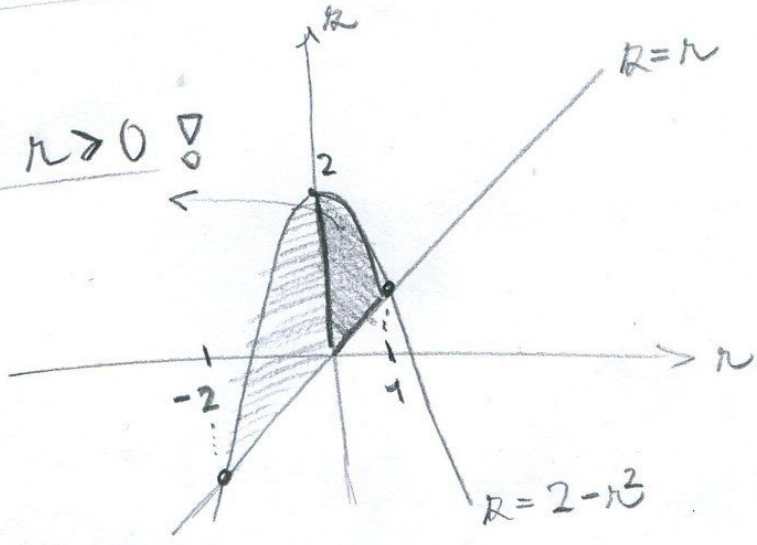
$r \le z \le 2-r^2$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$J = r$




zřejmě $r \ge 0$ !

průsečíky křivek:

$2-r^2 = r$

$r^2+r-2 = 0$

$(r+2)(r-1) = 0$

$r=-2 \vee r=1$

$r^2+r-2 \le 0$

$(r+2)(r-1) \le 0$

$\Leftrightarrow r \in \langle -2, 1\rangle$

| | $-\infty$ | $-2$ | $1$ | $\infty$ |
|---|---|---|---|---|
| $r-1$ | $-$ | $-$ | $0$ | $+$ |
| $r+2$ | $-$ | $0$ | $+$ | $+$ |
| | $+$ | $0$ | $-$ | $+$ |

$r \in (0,1)$

FUBINIOVA VĚTA ⊕ VĚTA O SUBSTITUCI

$$\mu(\Omega) = \int_\Omega 1 \, dx \overset{\text{FUB}}{=} \int_0^1 \int_0^{2\pi} \int_r^{2-r^2} r \, dz \, d\varphi \, dr =$$

$$= \int_0^1 \int_0^{2\pi} r\cdot(2-r^2-r)\, d\varphi \, dr = 2\pi \cdot \int_0^1 (2r - r^3 - r^2)\, dr$$

$$= 2\pi \cdot \left[ r^2 - \frac{r^4}{4} - \frac{r^3}{3} \right]_0^1 = 2\pi \cdot \left(1 - \frac{1}{4} - \frac{1}{3}\right) =$$

$$= 2\pi \cdot \frac{12-3-4}{12} = \pi \cdot \frac{5}{6} = \underline{\underline{\frac{5}{6}\pi}}$$