1a) $\int \frac{2x}{x^2-6x+13}dx = \int \frac{2x-6}{x^2-6x+13}dx + \int \frac{6}{x^2-6x+13}dx$

$$\left| \begin{array}{l} x^2-6x+13 = t \\ (2x-6)dx = dt \end{array} \right|$$

$\overset{(*)}{=} \ln(x^2-6x+13) + 3\operatorname{arctg}\left(\frac{x-3}{2}\right) + C$

1) $\int \frac{2x-6}{x^2-6x+13}dx = \int \frac{1}{t}dt = \ln|t| = \ln(x^2-6x+13)$

2) $\int \frac{6}{x^2-6x+13}dx = 6\cdot\int \frac{1}{x^2-6x+9+4}dx = 6\cdot\int \frac{1}{(x-3)^2+4}dx$

$= 6\cdot\frac{1}{4}\cdot\int \frac{1}{\left(\frac{x-3}{2}\right)^2+1}dx = \frac{3}{\cancel{2}}\cdot\cancel{2}\cdot\operatorname{arctg}\left(\frac{x-3}{2}\right)$

1b) $\int \frac{x^3}{x^2+2x-35}dx = \int\left(x-2+\frac{39x-70}{x^2+2x-35}\right)dx =$

$$x^3 : (x^2+2x-35) = x-2+\frac{39x-70}{x^2+2x-35}$$

$$\begin{array}{l} -(x^3+2x^2-35x) \\ \hline \quad -2x^2+35x \\ \quad -(-2x^2-4x+70) \\ \hline \quad\quad 39x-70 \end{array}$$

$= \frac{x^2}{2}-2x+\int\frac{\frac{343}{12}}{x+7}dx+\int\frac{\frac{125}{12}}{x-5}dx = \frac{x^2}{2}-2x+$
$+\frac{1}{12}\left(343\ln|x+7|+ +125\ln|x-5|\right)$

Rozklad na parciální zlomky:

$\frac{39x-70}{x^2+2x-35} = \frac{39x-70}{(x+7)(x-5)} = \frac{A}{x+7}+\frac{B}{x-5} \quad \Big|\cdot(x+7)(x-5)$

$$39x-70 = A(x-5)+B(x+7)$$

$x=5:\quad 125 = 0A+12B \implies B=\frac{125}{12}$

$x=-7:\quad -343 = -12A+0B \implies A=\frac{343}{12}$

1c) $\int \frac{1}{x^3+x^2-30x}\,dx = \int\left(\frac{\left(-\frac{1}{30}\right)}{x} + \frac{\frac{1}{66}}{x+6} + \frac{\frac{1}{55}}{x-5}\right)dx$

Rozklad na parciální zlomky:

$$\frac{1}{x^3+x^2-30x} = \frac{1}{x(x+6)(x-5)} = \frac{A}{x} + \frac{B}{x+6} + \frac{C}{x-5}$$

$$1 = A(x+6)(x-5) + B\cdot(x-5)\cdot x + Cx(x+6)$$

$x=0$: $1 = -30A \Rightarrow \boxed{A = -\frac{1}{30}}$

$x=5$: $1 = 55C \Rightarrow \boxed{C = \frac{1}{55}}$

$x=-6$: $1 = 66B \Rightarrow \boxed{B = \frac{1}{66}}$

$$= -\frac{1}{30}\ln|x| + \frac{1}{66}\ln|x+6| + \frac{1}{55}\ln|x-5| + c$$

1d) $\int_0^1 \operatorname{arctan} x\,dx = [x\cdot\operatorname{arctg} x]_0^1 - \int_0^1 \frac{*}{x^2+1}\,dx =$

PER-PARTES: $\int uv' = u\cdot v - \int u'v$ $\quad\left|\begin{matrix} u = \arctan x & v' = 1 \\ u' = \frac{1}{x^2+1} & v = x\end{matrix}\right|$

$$= 1\cdot\operatorname{arctg} 1 - 0\operatorname{arctg} 0 - \int_1^2 \frac{1}{t}\frac{1}{2}\,dt = \frac{\pi}{4} - \frac{1}{2}[\ln|t|]_1^2$$

$\left|\begin{matrix} x^2+1 = t \\ 2x\,dx = dt \end{matrix}\right|$

$x\,dx = \frac{1}{2}dt$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

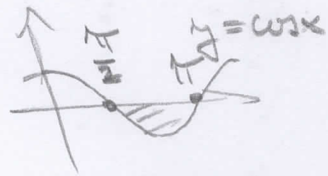
1e) $\displaystyle\int_0^1 \frac{x^3}{x^4+1}\,dx = \int_1^2 \frac{1}{t}\,\frac{1}{4}\,dt = \frac{1}{4}\Big[\ln|t|\Big]_1^2 = \frac{\ln 2}{4}$

$$\left| \begin{aligned} x^4+1 &= t \\ 4x^3\,dx &= dt \end{aligned} \right|$$

$$x^3\,dx = \frac{1}{4}\,dt$$

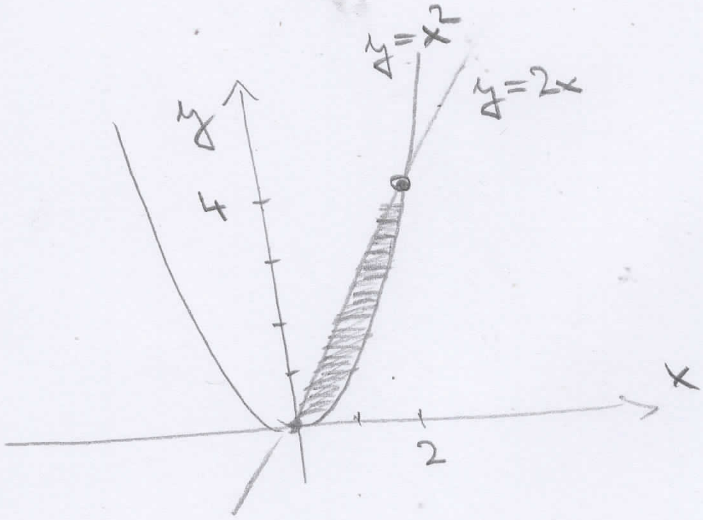
1f) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sqrt{\cos^3 x}\,\sin x\,dx \notin \mathbb{R}$, neboť integrand není definován na $\langle\frac{\pi}{2},\pi)$

Omlouvám se, chyba v zadání



$\forall x \in (\frac{\pi}{2},\pi): \cos x < 0$

2.



$$\int_0^2\int_{x^2}^{2x} 1\,dy\,dx = \int_0^2 \left(2x - x^2\right)dx = \left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$= 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$$