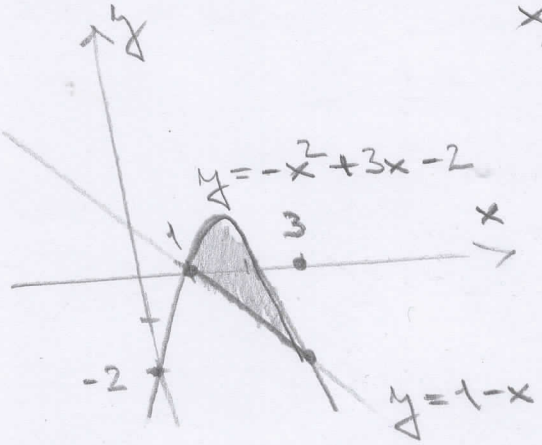
(1.) $1-x \le y \le -x^2+3x-2$

průsečíky křivek: $1-x = -x^2+3x-2$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3 \vee x=1$



$-x^2+3x-2 = -(x^2-3x+2)$

$= -(x-2)(x-1)$

Obsah plochy: $\int_1^3 \int_{1-x}^{-x^2+3x-2} 1 \, dy \, dx = \int_1^3 \left(-x^2+3x-2-(1-x)\right) dx$

$= \int_1^3 (-x^2+4x-3)\,dx = \left[-\frac{x^3}{3}+4\cdot\frac{x^2}{2}-3x\right]_1^3 =$

$= -9+2\cdot 9-9-\left(-\frac{1}{3}*2-3\right) = 0-\left(-\frac{4}{3}\right) = \frac{4}{3}$

(2.) Objem rotačního tělesa : $V = \pi \cdot \int_a^b f^2(x)\,dx$

průsečíky křivek : $6 - x = \frac{4}{x} + 1 \quad |-1$

$5 - x = \frac{4}{x} \quad |\cdot x$

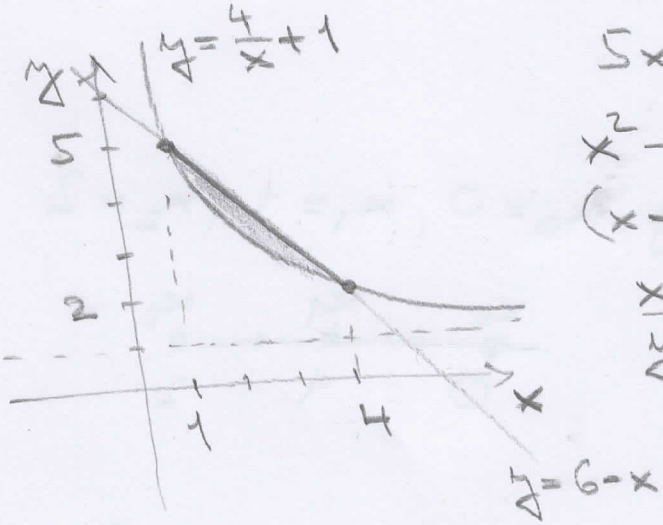
$5x - x^2 = 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1 \vee x = 4$

$y = 5 \qquad y = 2$



a) kolem osy x : $V = \pi \cdot \int_1^4 \left((6-x)^2 - \left(\frac{4}{x}+1\right)^2 \right) dx =$

$= \pi \cdot \int_1^4 \left(36 - 12x + x^2 - \frac{16}{x^2} - \frac{8}{x} - 1 \right) dx$

$= \pi \cdot \left[35x - 6x^2 + \frac{x^3}{3} + \frac{16}{x} - 8\ln|x| \right]_{-1}^{4}$

$= \pi \cdot \left(140 - 96 + \frac{64}{3} + 4 - 8\ln 4 - 35 + 6 - \frac{1}{3} - 16 \right)$

$= \pi \cdot (24 - 8\ln 4) \doteq 12{,}9\,\pi$

b) kolem osy y : $V = \pi \cdot \int_2^5 \left((6-y)^2 - \left(\frac{4}{y-1}\right)^2 \right) dy =$

$= \pi \cdot \int_2^5 \left(36 - 12y + y^2 - \frac{16}{(y-1)^2} \right) dy =$

$= \pi \cdot \left[36y - 6y^2 + \frac{y^3}{3} + \frac{16}{y-1} \right]_{-2}^{5} = \pi \cdot \left(180 - 150 + \frac{125}{3} + 4 - 72 + 24 + \frac{8}{3} - 16 \right)$

$= 9\pi$

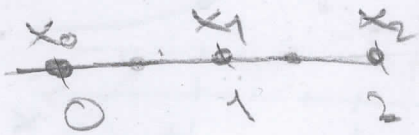
(3.) $\int_0^2 \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2 = \frac{2}{3}\cdot\sqrt{8} \doteq \underline{1{,}8856}$

↓ přesná hodnota

- $g(x) := \sqrt{x}$

Lichoběžníková metoda:

a) 2 dílky: uzlové body $x_0 = 0,\ x_1 = 1,\ x_2 = 2$



$g(0) = \sqrt{0} = 0$

$g(1) = \sqrt{1} = 1$

$g(2) = \sqrt{2} \doteq 1{,}41$

$$\int_0^2 \sqrt{x}\,dx \doteq \frac{1}{2}\cdot\left(g(0) + 2\cdot g(1) + g(2)\right)$$

$$= \frac{1}{2}(0 + 2\cdot 1 + \sqrt{2}) \doteq \underline{1{,}705}$$

chyba, kterou se dopustíme je přibližně

$$\left|1{,}8856 - 1{,}705\right| \doteq \underline{0{,}1806}$$

b) 4 dílky: uzlové body $x_0 = 0, x_1 = 0{,}5, x_2 = 1$
$x_3 = 1{,}5,\ x_4 = 2$

$$\int_0^2 \sqrt{x}\,dx \doteq \frac{0{,}5}{2}\cdot\left(g(0) + 2\left(g(0{,}5) + g(1) + g(1{,}5)\right) + g(2)\right)$$

$$= 0{,}25\cdot\left(0 + 2\cdot(0{,}707 + 1 + 1{,}22) + 1{,}41\right) \doteq \underline{1{,}816}$$

chyba: $\left|1{,}8856 - 1{,}816\right| \doteq \underline{0{,}0696}$