(1.) $y = 3e^{-5x} + 2e^{7x}$

$y' = -15e^{-5x} + 14e^{7x}$

$y'' = 75e^{-5x} + 98e^{7x}$

Dosazením do rovnice: $y'' - 2y' - 35y \stackrel{?}{=} 0$

$75e^{-5x} + 98e^{7x} - 2(-15e^{-5x} + 14e^{7x}) - 35(3e^{-5x} + 2e^{7x})$

$= 75e^{-5x} + 30e^{-5x} - 105e^{-5x} + 98e^{7x} - 28e^{7x} - 70e^{7x} = 0$

$\Rightarrow$ ano, je řešením dané ODR

(2.) $y' = \frac{1}{y^2 + y}$

$\frac{dy}{dx} = \frac{1}{y(x^2+1)}$ $\quad | \cdot y$ $\quad | \cdot dx$

$\int y\,dy = \int \frac{1}{x^2+1}\,dx$

$\frac{y^2}{2} = \operatorname{arctg} x + c$

$y(0) = -\sqrt{2}$: $1 = \operatorname{arctg} 0 + c \Rightarrow c = 1$

$y = -\sqrt{2\operatorname{arctg} x + 2}$

$2\operatorname{arctg} x + 2 \geq 0$

$\operatorname{arctg} x \geq -1$

$D_f = \langle \operatorname{tg}(-1), +\infty)$

LDR 1. ŘÁDU S KONSTANTNÍMI KOEFICIENTY

(3.) $y' + y\cos x = \sin 2x \quad | \cdot e^{P(x)}$

$y' + y f(x) = g(x)$

INTEGRAČNÍ FAKTOR

KDE $P(x) = \int f(x) = \int \cos x = \sin x$

$y' \cdot e^{\sin x} + y \cos x \cdot e^{\sin x} = e^{\sin x} \cdot \sin 2x$

$(y \cdot e^{\sin x})' = e^{\sin x} \cdot \sin 2x$

$y \cdot e^{\sin x} = \int e^{\sin x} \cdot \sin 2x \, dx$

$\int e^{\sin x} \cdot \sin 2x \, dx = \int e^{\sin x} \cdot 2 \sin x \cos x \, dx =$

$= \left| \begin{array}{l} \sin x = t \\ \cos x \, dx = dt \end{array} \right| \int e^t \cdot 2t \, dt = e^t \cdot 2t - \int 2e^t \, dt$

PER-PARTES: $\left| \begin{array}{ll} u = 2t & v' = e^t \\ u' = 2 & v = e^t \end{array} \right|$

$= 2 \sin x e^{\sin x} - 2e^{\sin x}$

$\Rightarrow y \cdot e^{\sin x} = 2 \sin x e^{\sin x} - 2e^{\sin x} + c \quad | : e^{\sin x}$

$y = 2 \sin x - 2 + c \cdot e^{-\sin x}$

$y(0) = 5: \quad 5 = 2 \sin 0 - 2 + c \cdot e^0 \Rightarrow c = 7$

$y = 2 \sin x - 2 + 7 e^{-\sin x}$