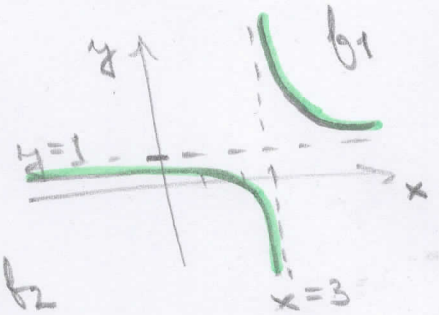
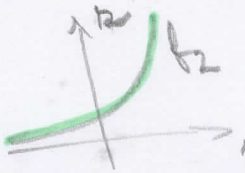
1.1. $f(x) = 2^{\frac{x-2}{x-3}} = (f_2 \circ f_1)(x)$

$f_1(x) = \frac{x-2}{x-3}$ - prostá funkce

$f_2(y) = 2^y$ - prostá funkce




$\Rightarrow$ $f(x) = (f_2 \circ f_1)(x)$ je složením prostých funkcí $\Rightarrow$ je prostá,

tedy existuje $f^{-1}$

$f$: $y = 2^{\frac{x-2}{x-3}}$

$\log_2 y = \frac{x-2}{x-3} \quad | \cdot (x-3)$

$(x-3)\log_2 y = x-2$

$x\log_2 y - 3\log_2 y = x-2$

$x\log_2 y - x = 3\log_2 y - 2$

$x(\log_2 y - 1) = 3\log_2 y - 2 \quad | : (\log_2 y - 1)$

$x = \frac{3\log_2 y - 2}{\log_2 y - 1}$

$f^{-1}: y = \frac{3\log_2 x - 2}{\log_2 x - 1}$

$D_f = \mathbb{R} - \{3\} = H_{f^{-1}}$

$\mathbb{R} - \{3\} \xrightarrow{f_1} \mathbb{R} - \{1\} \xrightarrow{f_2} (0,2) \cup (2,\infty)$

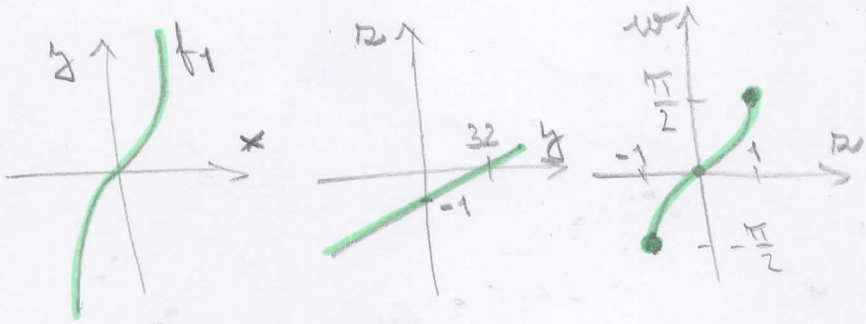
$H_f = D_{f^{-1}} = (0,2) \cup (2,\infty)$

1.2. $f(x) = \arcsin\left(\frac{x^3}{32} - 1\right) = (f_3 \circ f_2 \circ f_1)(x)$

$f_1(x) = x^3$

$f_2(y) = \frac{y}{32} - 1$

$f_3(z) = \arcsin z$




$f_1, f_2, f_3$ jsou prosté $\Rightarrow$ f je prostá $\rightarrow \exists f^{-1}$

$$-1 \leq \frac{x^3}{32} - 1 \leq 1 \quad /+1$$

$$0 \leq \frac{x^3}{32} \leq 2 \quad /\cdot 32$$

$$0 \leq x^3 \leq 64$$

$$0 \leq x \leq 4$$

$$\boxed{D_f = \langle 0,4 \rangle} = H_{f^{-1}}$$

$f: y = \arcsin\left(\frac{x^3}{32} - 1\right)$

$$\sin y = \frac{x^3}{32} - 1$$

$$\sin y + 1 = \frac{x^3}{32}$$

$$32(\sin y + 1) = x^3$$

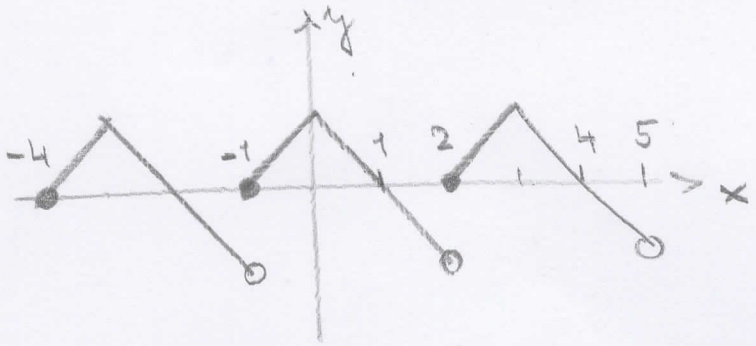
$$\sqrt[3]{32(\sin y + 1)} = x$$

$$\Rightarrow \boxed{f^{-1}: y = \sqrt[3]{32(\sin x + 1)}}$$

$$\langle 0,4 \rangle \xrightarrow{f_1} \langle 0,64 \rangle \xrightarrow{f_2} \langle -1,1 \rangle \xrightarrow{f_3} \langle -\tfrac{\pi}{2}, \tfrac{\pi}{2} \rangle$$

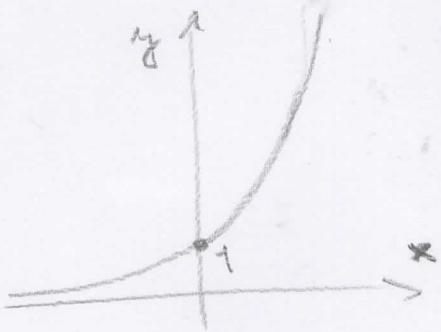
$$\boxed{H_f = D_{f^{-1}} = \langle -\tfrac{\pi}{2}, \tfrac{\pi}{2} \rangle}$$

2.




Funkce g není spojitá v bodech tvaru $x = 2 + 3k, k \in \mathbb{Z}$

3.




např. $f(x) = 2^x$

4.