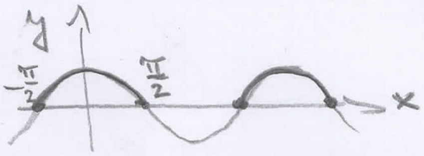
1.

1. $f(x) = \ln(\cos x)$ $\qquad \cos x > 0 \iff x \in \left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$

$k \in \mathbb{Z}$

$$f'(x) = \frac{1}{\cos x} \cdot (-\sin x)$$



$$= -\frac{\sin x}{\cos x} = \underline{\underline{-\operatorname{tg} x}}$$

$$\boxed{D_f = \left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)}$$

$k \in \mathbb{Z}$

2. $f(x) = \ln(\sin 3x)$ $\qquad \sin 3x > 0 \iff 3x \in (0 + 2k\pi, \pi + 2k\pi)$

$$x \in \left(\frac{2k\pi}{3}, \frac{\pi + 2k\pi}{3}\right)$$

$k \in \mathbb{Z}$

$$f'(x) = \frac{1}{\sin 3x} \cdot \cos 3x \cdot 3$$

$$= 3 \cdot \frac{\cos 3x}{\sin 3x} = \underline{\underline{3 \operatorname{ctg} 3x}}$$

$$\boxed{D_f = \left(\frac{2}{3}k\pi, \frac{\pi}{3} + \frac{2}{3}k\pi\right)} \quad k \in \mathbb{Z}$$

3. $f(x) = \ln\left(\frac{2-x}{2+x}\right)$

$$\left(\frac{a}{b}\right)' = \frac{a' \cdot b - a \cdot b'}{b^2}$$

$$f'(x) = \frac{1}{\frac{2-x}{2+x}} \cdot \frac{(-1)(2+x) - (2-x) \cdot 1}{(2+x)^2} = \frac{-2-x-2+x}{(2-x)(2+x)} =$$

$$= \frac{-4}{4-x^2} = \underline{\underline{\frac{4}{x^2-4}}}$$

$$\frac{2-x}{2+x} > 0 \iff x \in (-2, 2)$$

	$-\infty$ … -2	-2 … 2	2 … ∞
$2-x$	+	+	0 −
$2+x$	− 0	+	+
	−	+	−

$$\boxed{D_f = (-2, 2)}$$

4. $f(x) = \operatorname{arctg}\left(\frac{x+1}{x-1}\right)$ $\qquad \boxed{D_f = \mathbb{R} \setminus \{1\}}$

$$f'(x) = \frac{1}{1 + \left(\frac{x+1}{x-1}\right)^2} \cdot \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2 + (x+1)^2} =$$

$$= -\frac{2}{x^2-2x+1+x^2+2x+1} = -\frac{2}{2x^2+2} = -\frac{\cancel{2}}{\cancel{2}(x^2+1)} = \underline{\underline{-\frac{1}{x^2+1}}}$$

5. $f(x)=\sqrt{\sin^2(x^3)+1}$

$$f'(x)=\frac{1}{2}\cdot\left(\sin^2(x^3)+1\right)^{-\frac{1}{2}}\cdot 2\sin(x^3)\cdot\cos(x^3)\cdot 3x^2 =$$

$$=\frac{3x^2\sin(x^3)\cos(x^3)}{\sqrt{\sin^2(x^3)+1}}$$

D_f: $\sin^2(x^3)+1\geq 0 \quad \forall x\in\mathbb{R}$

neboť $y^2+1\geq 0 \ \forall y\in\mathbb{R}$

$$\boxed{D_f=\mathbb{R}}$$

2.

1. $$\lim_{x\to 0}\frac{\sin 2x}{3x}\underset{"\frac{0}{0}"}{\overset{L.P.}{=}}\lim_{x\to 0}\frac{2\cos 2x}{3}=\frac{2\cos 0}{3}=\underline{\underline{\frac{2}{3}}}$$

2. $$\lim_{x\to\infty}x^2e^{-x}=\lim_{x\to\infty}\frac{x^2}{e^x}\underset{"\frac{\infty}{\infty}"}{\overset{L.P.}{=}}\lim_{x\to\infty}\frac{2x}{e^x}\underset{"\frac{\infty}{\infty}"}{\overset{L.P.}{=}}\lim_{x\to\infty}\frac{2}{e^x}=\underline{\underline{0}}$$

3. $$\lim_{x\to 0}\frac{1-\cos 6x}{x^2}\underset{"\frac{0}{0}"}{\overset{L.P.}{=}}\lim_{x\to 0}\frac{6\cdot\sin 6x}{2x}\underset{"\frac{0}{0}"}{\overset{L.P.}{=}}\lim_{x\to 0}\frac{6\cdot 6\cdot\cos 6x}{2}=\underline{\underline{18}}$$

4. $$\lim_{x\to 0_+}\frac{1-e^{\sin 5x}}{\ln(1+\sin x)}\underset{"\frac{0}{0}"}{\overset{L.P.}{=}}\lim_{x\to 0_+}\frac{-e^{\sin 5x}\cdot\cos 5x\cdot 5}{\frac{1}{1+\sin x}\cdot\cos x}=$$

$$=\frac{-e^{\sin 0}\cdot\cos 0\cdot 5}{\frac{1}{1+\sin 0}\cdot\cos 0}=\frac{-5}{\frac{1}{1+0}\cdot 1}=\underline{\underline{-5}}$$

5. $$\lim_{x\to 0} \frac{e^{x^2}-1}{x\sin(7x)} \overset{\text{L.P.}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to 0} \frac{2x\,e^{x^2}}{\sin(7x) + x\cdot\cos(7x)\cdot 7}$$

$$\overset{\text{L.P.}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to 0} \frac{2e^{x^2} + (2x)\cdot(2x)\cdot e^{x^2}}{\cos(7x)\cdot 7 + 7\cos(7x) + 7x\cdot 7\cdot(-\sin 7x)}$$

$$= \frac{2e^0 + 0}{\cos 0\cdot 7 + 7\cos 0 + 0} = \frac{2}{7+7} = \frac{2}{14} = \underline{\underline{\frac{1}{7}}}$$

3. $f(x) = -x^2 + 4x + 5$ $\quad x=1 \quad f(1) = -1+4+5 = 8$

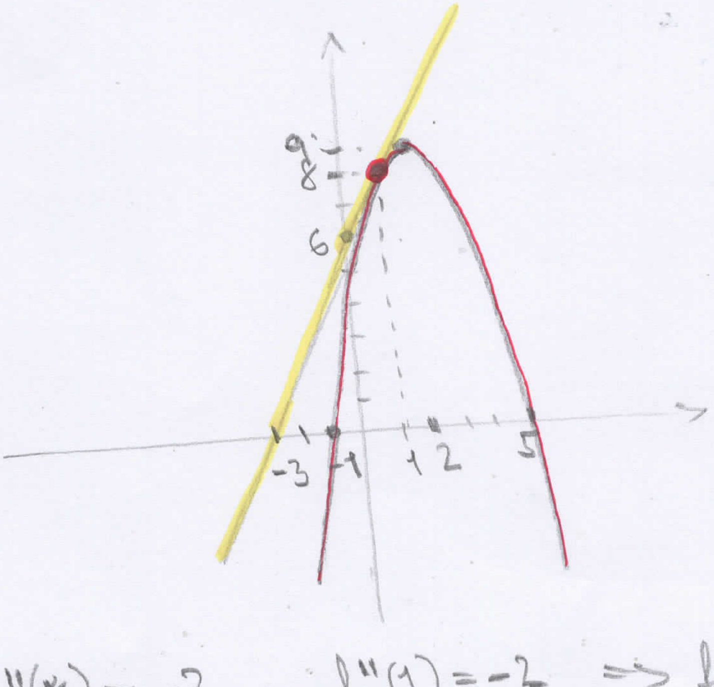
$f'(x) = -2x + 4$ $\quad f'(1) = 2 = k$

tečna k funkci f v bodě [1;8] je t: $y = \underset{\overset{\parallel}{2}}{k}x + q$

$[1,8] \in t \implies 8 = 2\cdot 1 + q$

$q = 6$

$$\boxed{t: y = 2x + 6}$$



$f(x) = -x^2 + 4x + 5$

$= -(x^2 - 4x - 5)$

$= -(x-5)(x+1)$

Vrchol: [2;9]

$f''(x) = -2$, $f''(1) = -2 \implies f$ je konkávní v [1;8]