(1.) $f(x) = x^3 - 6x^2 + 9x$ $\qquad D_f = \mathbb{R}$

1. $f(x) = 0 \iff x^3 - 6x^2 + 9x = 0$

$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)^2 = 0$$
$$\underline{x=0} \vee \underline{x=3}$$

$f(0) = 0^3 - 6\cdot 0^2 + 9\cdot 0 = 0$

průsečíky s osami: $[0;0]$, $[3;0]$

2. $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$

$f'(x) = 0 \iff \underline{x=3} \vee \underline{x=1}$

$\nearrow \quad \searrow \quad \nearrow$

$f'>0 \quad 1 \quad f'<0 \quad 3 \quad f'>0$

$\forall x \in (1,3): f'(x) < 0 \Rightarrow f$ je klesající na $(1,3)$

$\forall x \in (-\infty,1) \cup (3,\infty): f'(x) > 0$

$\Rightarrow f$ je rostoucí na $(-\infty,1)$ a $(3,\infty)$

$f(1) = 1^3 - 6\cdot 1^2 + 9\cdot 1 = 4$ $\qquad$ lok. maximum: $[1;4]$

$f(3) = 3^3 - 6\cdot 3^2 + 9\cdot 3 = 27 - 54 + 27 = 0$

lok. minimum: $[3;0]$

3. $f''(x) = 6x - 12$

$f''(x) = 0 \iff 6x - 12 = 0 \quad | +12$

$$6x = 12 \quad | :6$$
$$x = 2$$

$\frown \quad \smile$

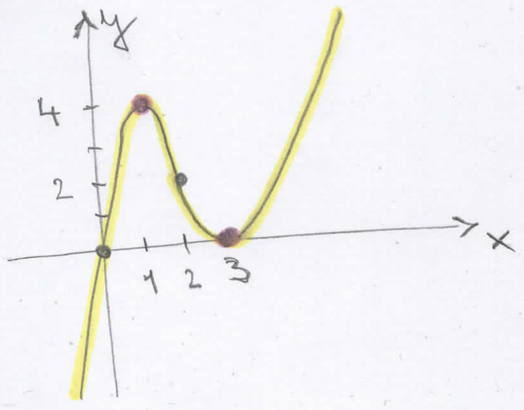
$f''<0 \quad 2 \quad f''>0$

$\forall x \in (2,\infty): f''(x) > 0 \Rightarrow f$ je konvexní

$\forall x \in (-\infty,2): f''(x) < 0 \Rightarrow f$ je konkávní

$f(2) = 2^3 - 6\cdot 2^2 + 9\cdot 2 = 8 - 24 + 18 = 2$

inflexní bod: $[2;2]$



(2.) $f(x) = \frac{1}{x} + \ln x$ $D_f = (0, \infty)$

1. derivace : $f'(x) = -\frac{1}{x^2} + \frac{1}{x} = \frac{(-1)+x}{x^2} = \frac{x-1}{x^2}$

$f'(x) = 0 \iff \frac{x-1}{x^2} = 0 \quad | \cdot x^2$

$x - 1 = 0$

$x = 1$

0 $f'<0$ 1 $f'>0$

$\forall x \in (0,1): f'(x) < 0 \Rightarrow f$ klesá

$\forall x \in (1,\infty): f'(x) > 0 \Rightarrow f$ roste

$f(1) = \frac{1}{1} + \ln 1 = 1 + 0 = 1$ lok. minimum : $[1;1]$

2. derivace : $f''(x) = \left(\frac{x-1}{x^2}\right)' = \frac{1 \cdot x^2 - (x-1) \cdot 2x}{(x^2)^2} = \frac{-x^2+2x}{x^4} = \frac{2-x}{x^3}$

$\left(\frac{a}{b}\right)' = \frac{a' \cdot b - a \cdot b'}{b^2}$

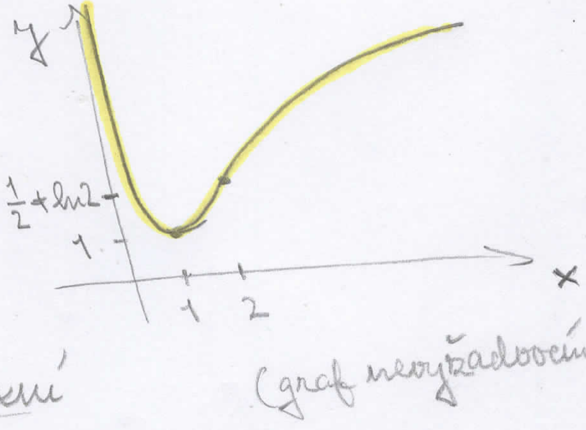
DERIVACE PODÍLU

$f''(x) = 0 \iff \frac{2-x}{x^3} = 0$

$x = 2$

$f(2) = \frac{1}{2} + \ln 2$ - inflexní bod

0 $f''>0$ 2 $f''<0$



(graf nevyžadovaný)

$\forall x \in (0,2): f''(x) > 0 \Rightarrow f$ je konvexní

$\forall x \in (2,\infty): f''(x) < 0 \Rightarrow f$ je konkávní

(3.) $f(x) = \operatorname{arctan}\left(\frac{1}{x}\right)$ $\qquad D_f = \mathbb{R} - \{0\}$

1) $f'(x) = \dfrac{1}{\left(\frac{1}{x}\right)^2+1} \cdot \left(-\dfrac{1}{x^2}\right) = -\dfrac{1}{1+x^2}$

$\forall x \in D_f : f'(x) < 0 \Rightarrow$ f je klesající na $(-\infty, 0)$
a na $(0, \infty)$

$\forall x \in D_f : f'(x) \neq 0 \Rightarrow$ f nemá extrém
(a $f \in C^1(D_f)$)

2) $f''(x) = \left(-\dfrac{1}{1+x^2}\right)' = \dfrac{2x}{(1+x^2)^2}$

$f''(x) = 0 \Leftrightarrow x = 0 \notin D_f \Rightarrow$ f nemá inflexní bod

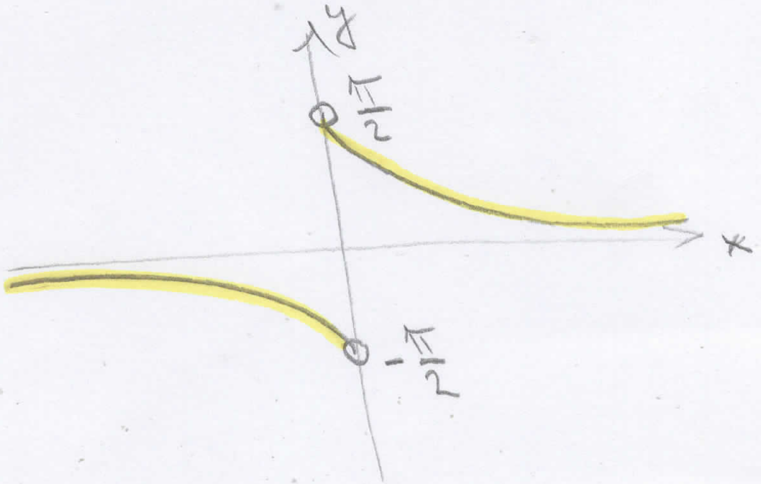
$f'' < 0 \quad 0 \quad f'' > 0$

$\forall x \in (0, \infty) : f''(x) > 0 \Rightarrow$ f je konvexní
$\forall x \in (-\infty, 0) : f''(x) < 0 \Rightarrow$ f je konkávní

3) $\lim\limits_{x \to \pm\infty} \operatorname{arctan}\left(\frac{1}{x}\right) = \operatorname{arctan} 0 = 0$

$\lim\limits_{x \to 0_+} \operatorname{arctan}\left(\frac{1}{x}\right) = \operatorname{arctan}\left(\frac{1}{0_+}\right) = \frac{\pi}{2}$

$\lim\limits_{x \to 0_-} \operatorname{arctan}\left(\frac{1}{x}\right) = \operatorname{arctan}\left(\frac{1}{0_-}\right) = -\frac{\pi}{2}$



4. $f(x) = \dfrac{6-10x}{5x-30}$ $\qquad D_f = \mathbb{R} \setminus \{6\}$

$$\lim_{x \to 6_+} f(x) = \frac{-54}{0_+} = -\infty$$

$$\lim_{x \to 6_-} f(x) = \frac{-54}{0_-} = +\infty$$

$\Rightarrow$ přímka $x=6$ je svislá asymptota

$$\lim_{x \to \pm\infty} \frac{6-10x}{5x-30} = \lim_{x \to \pm\infty} \frac{\cancel{x}\left(\frac{6}{x}-10\right)}{\cancel{x}\left(5-\frac{30}{x}\right)} = \frac{0-10}{5-0} = -2$$

$\Rightarrow$ přímka $y=-2$ je vodorovná asymptota