(1.) $F_1(x) = \dfrac{1+2x}{x} = \dfrac{1}{x} + 2$

$F_2(x) = \dfrac{1-x}{x} = \dfrac{1}{x} - 1$

Ano, F_1, F_2 jsou primitivními funkcemi k téže funkci na $(0,\infty)$, neboť se liší pouze o aditivní konstantu,

tj. $\boxed{\exists c \in \mathbb{R} : F_1 = F_2 + c}$, zde $c = 3$

nebo stačí ověřit, že $F_1' = F_2'$

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$$

(2) a) $$\int \frac{3x+1}{\sqrt{x}}\, dx = 3\cdot\int x^{\frac{1}{2}}\, dx + \int x^{-\frac{1}{2}}\, dx = 3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= \cancel{3}\cdot\frac{2}{\cancel{3}}\cdot\sqrt{x^3} + 2\sqrt{x} = \underline{2\sqrt{x^3} + 2\sqrt{x} + c}$$

$x \in (0,\infty)$

b) $$\int x e^{3x}\, dx = \frac{1}{3}x e^{3x} - \int 1\cdot\frac{1}{3}e^{3x}\, dx = \underline{e^{3x}\cdot\left(\frac{1}{3}x - \frac{1}{9}\right) + c}$$

$x \in \mathbb{R}$

PER-PARTES: $\boxed{\int u v' = u\cdot v - \int u' v}$

$$\begin{vmatrix} u = x & v' = e^{3x} \\ u' = 1 & v = \frac{1}{3}e^{3x} \end{vmatrix}$$

c) $$\int \ln(2x)\cdot x^3\, dx = \frac{x^4}{4}\cdot\ln(2x) - \int \frac{1}{x}\cdot\frac{x^4}{4}\, dx$$

$$= \frac{x^4}{4}\cdot\ln(2x) - \frac{1}{4}\cdot\frac{x^4}{4} + c$$

PER-PARTES

$$\begin{vmatrix} u = \ln(2x) & v' = x^3 \\ u' = \frac{1}{x} & v = \frac{x^4}{4} \end{vmatrix}$$

$$= \underline{\frac{x^4}{4}\cdot\left(\ln(2x) - \frac{1}{4}\right) + c}$$

$x \in (0,\infty)$

d) $\int \frac{100}{25+9x^2}\,dx = 100\cdot\frac{1}{25}\cdot\int\frac{1}{1+\frac{9}{25}x^2}\,dx =$

$= 4\cdot\int\frac{1}{1+\left(\frac{3}{5}x\right)^2}\,dx = \underline{4\cdot\frac{5}{3}\cdot \operatorname{arctg}\left(\frac{3}{5}x\right)+c}$

$x\in\mathbb{R}$

$\left|\begin{array}{l}\frac{3}{5}x = t\\ \frac{3}{5}\,dx = dt\end{array}\right|$

e) $\int e^{-x^5}\cdot 15x^4\,dx = \int e^{t}\cdot 15\cdot\left(-\frac{1}{5}\right)dt = -3\int e^{t}\,dt$

$= \underline{-3e^{-x^5}+c}$

$x\in\mathbb{R}$

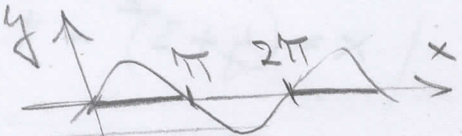
substituce: $\left|\begin{array}{l}-x^5 = t\\ -5x^4\,dx = dt\\ x^4\,dx = \left(-\frac{1}{5}\right)dt\end{array}\right|$

f) $\int\sqrt{\sin^5 x}\,\cos x\,dx = \int\sqrt{t^5}\,dt = \int t^{\frac{5}{2}}\,dt =$

substituce: $\left|\begin{array}{l}\sin x = t\\ \cos x\,dx = dt\end{array}\right|$

$= \frac{t^{\frac{7}{2}}}{\frac{7}{2}} = \underline{\frac{2}{7}\cdot\sqrt{\sin^7 x}+c}$

$\sin^5 x \ge 0 \iff \sin x \ge 0$



$\boxed{x\in\langle 2k\pi,\ 2k\pi+\pi\rangle}$, $k\in\mathbb{Z}$