DÚ - MA 1

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1. $A \cdot \vec{x} = \lambda \cdot \vec{x}$

$\det(A - \lambda E) = 0$

$$A = \begin{pmatrix} 5 & 2 \\ 8 & 5 \end{pmatrix} \Rightarrow \det\left(\begin{pmatrix} 5 & 2 \\ 8 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = \det \begin{pmatrix} 5-\lambda & 2 \\ 8 & 5-\lambda \end{pmatrix} = 0$$

$$(5-\lambda)^2 - 2 \cdot 8 = 0$$

$$25 - 10\lambda + \lambda^2 - 16 = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$(\lambda - 1)(\lambda - 9) = 0$$

$\lambda_1 = 1$; $\lambda_2 = 9$

(2 vlastní čísla)

$\lambda_1 = 1$:

$$\operatorname{Ker} \begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\rangle$$

$\lambda_2 = 9$:

$$\operatorname{Ker} \begin{pmatrix} -4 & 2 \\ 8 & -4 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\rangle$$

$$B = \begin{pmatrix} 3 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow \det\left(\begin{pmatrix} 3 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) = \det \begin{pmatrix} 3-\lambda & -2 & 1 \\ 0 & -1-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0$$

det. HT-matice je roven součinu prvků na diagonále:

$$(3-\lambda)(-1-\lambda)(4-\lambda) = 0$$

$\lambda_1 = 3$

$\lambda_2 = -1$

$\lambda_3 = 4$

$\lambda_1 = 3$:

$$\operatorname{Ker} \begin{pmatrix} 0 & -2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

$\lambda_2 = -1$:

$$\operatorname{Ker} \begin{pmatrix} 4 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\rangle$$

$\lambda_3 = 4$:

$$\operatorname{Ker} \begin{pmatrix} -1 & -2 & 1 \\ 0 & -5 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \left\langle \begin{pmatrix} \frac{3}{5} \\ \frac{1}{5} \\ 1 \end{pmatrix} \right\rangle$$

2.

$$\begin{vmatrix} 2 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ -2 & 1 & 2 & -2 \\ 2 & 2 & -2 & 1 \end{vmatrix} \overset{\text{rozvoj det. dle 1. řádku}}{=} 2 \cdot (-1)^{(1+1)} \cdot \left| \begin{array}{ccc} -2 & 0 & 0 \\ 1 & 2 & -2 \\ 2 & -2 & 1 \\ \hdashline -2 & 0 & 0 \\ 1 & 2 & -2 \end{array} \right| = 2 \cdot \left[-4 + 0 + 0 - (0 - 8 + 0) \right] =$$

$$\underline{\underline{= 8}}$$

3.

$$A = \left(\begin{array}{ccc|c} 1 & 3 & -3 & 3 \\ 4 & 2 & -1 & k \\ 3 & -1 & 2 & 3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -3 & 3 \\ 0 & -10 & 11 & k-12 \\ 0 & -10 & 11 & -6 \end{array} \right)$$

$-4 \cdot 1 + 2 = 2$

$-3 \cdot 1 + 3 = 3$

$$\Downarrow$$

$$k - 12 = -6$$

$$\boxed{k = 6} \; : \; \left(\begin{array}{ccc|c} 1 & 3 & -3 & 3 \\ 0 & -10 & 11 & -6 \end{array} \right)$$

$\underline{z = t}$

$-10y + 11t = -6$

$10y = 11t + 6$

$\underline{y = \frac{11}{10} t + \frac{3}{5}}$

$x + \frac{33}{10} t + \frac{15}{5} - 3t = 3$

$x = \frac{30}{10} t - \frac{33}{10} t - 3 + 3$

$\underline{x = -\frac{3}{10} t}$

$\Rightarrow$ ŘEŠENÍ:

$$\left\{ t \begin{pmatrix} -\frac{3}{10} \\ \frac{11}{10} \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{3}{5} \\ 0 \end{pmatrix} ; t \in \mathbb{R} \right\}$$

4.) matice A je singulární $\Leftrightarrow$ $\det A = 0$

$$\left|\begin{array}{ccc} a & 0 & 8 \\ 14 & a & 23 \\ 2 & 0 & a \\ \hline a & 0 & 8 \\ 14 & a & 23 \end{array}\right| = a^3 + 0 + 0 - (16a + 0 + 0) = a^3 - 16a = 0$$

$$a(a^2 - 16) = 0$$

$$\Downarrow$$

$a_1 = 0$

$a_2 = 4$

$a_3 = -4$