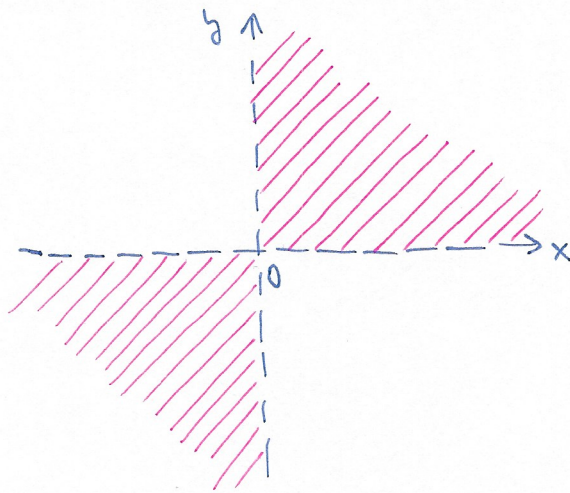


DÚ-MAT ④

DENISA VONDROVA'

① $f(x,y) = 2 \ln(xy) - x^2 - y$

a) $x \cdot y > 0$



b) ① STAC. BODY

$$\frac{df}{dx}(x,y) = \frac{2y}{xy} - 2x = \frac{2}{x} - 2x$$

$$\frac{2}{x} - 2x = 0 \Rightarrow 2 = 2x^2 \Rightarrow x = +1$$

$$x = -1$$

$$\frac{df}{dy}(x,y) = \frac{2x}{xy} - 1 = \frac{2}{y} - 1$$

$$\frac{2}{y} - 1 = 0 \Rightarrow y = 2$$

$$S_1 = [1; 2]$$

$$S_2 = [-1; 2] \text{ není v } D_f$$

② EXTREMÝ / SEDLOVÉ BODY

$$\frac{d^2f}{dx^2}(x,y) = \frac{-2}{x^2} - 2$$

HESSOVA MATICE :

$$\begin{pmatrix} \frac{d^2f}{dx^2}(x,y) & \frac{d^2f}{dydx}(x,y) \\ \frac{d^2f}{dxdy}(x,y) & \frac{d^2f}{dy^2}(x,y) \end{pmatrix} = \begin{pmatrix} \frac{-2}{x^2} - 2 & 0 \\ 0 & -\frac{2}{y^2} \end{pmatrix}$$

$$\frac{d^2f}{dxdy}(x,y) = \frac{df}{dydx}(x,y) = 0$$

$$\frac{d^2f}{dy^2}(x,y) = -\frac{2}{y^2}$$

$$S_1: H_f(1;2) = \begin{bmatrix} -4 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = 2 > 0 \Rightarrow \text{lok. extrém}$$

$$\frac{d^2f}{dx^2}(1,2) = -4 < 0 \Rightarrow \text{funkce má v } (1,2) \text{ lokální maximum ; } f(1,2) = 2 \ln 2 - 3$$

② $f(x,y) = x^3 + y^2 - 6xy - 39x + 18y + 20$ $D_f \in \mathbb{R}^2$

① STAC. BODY

$\frac{df}{dx}(x,y) = 3x^2 - 6y - 39$

$3x^2 - 6y - 39 = 0$

$\frac{df}{dy}(x,y) = 2y - 6x + 18$

$2y - 6x + 18 = 0 \Rightarrow x = \frac{1}{3}y + 3$

$\Rightarrow 3\left(\frac{1}{3}y + 3\right)^2 - 6y - 39 = 0$

$3\left(\frac{1}{9}y^2 + 2y + 9\right) - 6y - 39 = 0$

$\frac{1}{3}y^2 + 6y + 27 - 6y - 39 = 0$

$\frac{1}{3}y^2 - 12 = 0$

$y^2 = 36$

$y = 6, y = -6$

$y = 6$:

$3x^2 - 36 - 39 = 0$

$x^2 = 25$

$x = 5$, ~~$x = -5$~~

$12 - 6x + 18 = 0$

$x = 5$

$y = -6$:

$3x^2 + 36 - 39 = 0$

$x^2 = 1$

$x = 1$, ~~$x = -1$~~

$-12 - 6x + 18 = 0$

$x = 1$ ✓

$S_1 = [5; 6]$ $S_2 = [1; -6]$

② EXTREMÝ / SEDLOVÉ BODY

$\frac{d^2f}{dx^2}(x,y) = 6x$

HESSOVA MATICE: $\begin{pmatrix} 6x & -6 \\ -6 & 2 \end{pmatrix}$

$\frac{d^2f}{dx dy}(x,y) = \frac{d^2f}{dy dx}(x,y) = -6$

$\frac{d^2f}{dy^2}(x,y) = 2$

$S_1: H_f(5;6) = \begin{bmatrix} 30 & -6 \\ -6 & 2 \end{bmatrix} = 24 > 0 \Rightarrow$ *fre ma' v (5;6) extrem*
 $\frac{d^2f}{dx^2}(5,6) = 30 > 0 \Rightarrow$ *LOK. MINIMUM,*
 $f(5;6) = -86$

$S_2: H_f(1;-6) = \begin{bmatrix} 6 & -6 \\ -6 & 2 \end{bmatrix} = -24 < 0 \Rightarrow$ *fre ma' v (1;-6) sedlov' bod,*
 $f(1;-6) = -54$

③

x	-3	-2	0	1	4
y	-1	-1,5	0	1	2

→ aproksimoval fu' $y = ax + b$

a) → nait koeficienty a, b
a, b musit ricit soustavu

$$\left(\sum_{i=1}^5 x_i^2 \right) a + \left(\sum_{i=1}^5 x_i \right) b = \sum_{i=1}^5 x_i y_i$$

$$\left(\sum_{i=1}^5 x_i \right) a + 5b = \sum_{i=1}^5 y_i$$

$$\sum_{i=1}^5 x_i = -3 - 2 + 0 + 1 + 4 = 0$$

$$\sum_{i=1}^5 x_i^2 = 3^2 + 2^2 + 0^2 + 1^2 + 4^2 = 30$$

$$\sum_{i=1}^5 x_i y_i = 3 + 3 + 0 + 1 + 8 = 15$$

$$\sum_{i=1}^5 y_i = -1 - 1,5 + 0 + 1 + 2 = 0,5$$

$$\Rightarrow 30a + 0b = 15$$

$$\Rightarrow a = 0,5$$

$$0a + 5b = 0,5$$

$$\Rightarrow b = 0,1$$

$$y = 0,5x + 0,1$$

