① $F(x,y,z) = x^3 + y^3 + z^3 - 2\sqrt{z} = 0 \qquad (1,0,1)$

<u>OVĚŘENÍ</u> $\quad F(1,0,1) = 0$ ✓ $\qquad \frac{\partial F}{\partial z}(x,y,z) = 3z^2 - \frac{1}{\sqrt{z}}$

$$\frac{\partial F}{\partial z}(1,0,1) = 2 \neq 0 \text{ ✓}$$

$\Rightarrow$ rovnice implicitně def. fci $z = h(x,y)$ v okolí $(1,0,1)$

<u>DERIVACE (1. ZPŮSOB)</u>

$$x^3 + y^3 + h^3(x,y) - 2\sqrt{h(x,y)} = 0$$

$\frac{\partial}{\partial x}$: $\quad 3x^2 + 3h^2(x,y)\frac{\partial h}{\partial x}(x,y) - \frac{1}{\sqrt{h(x,y)}} \cdot \frac{\partial h}{\partial x}(x,y) = 0$

$$3 + 3\frac{\partial h}{\partial x}(1,0) - \frac{\partial h}{\partial x}(1,0) = 0 \qquad \boxed{\frac{\partial h}{\partial x}(1,0) = -\frac{3}{2}}$$

$\frac{\partial}{\partial y}$: $\quad 3y^2 + 3h^2(x,y)\frac{\partial h}{\partial y}(x,y) - \frac{1}{\sqrt{h(x,y)}} \cdot \frac{\partial h}{\partial y}(x,y) = 0$

$$0 + 3\frac{\partial h}{\partial y}(1,0) - \frac{\partial h}{\partial y}(1,0) = 0 \qquad \boxed{\frac{\partial h}{\partial y}(1,0) = 0}$$

$$\boxed{\operatorname{grad} h(1,0) = \left(-\frac{3}{2}, 0\right)}$$

<u>DERIVACE (2. ZPŮSOB)</u>

$\frac{\partial F}{\partial x}(x,y,z) = 3x^2 \qquad \frac{\partial h}{\partial x}(x,y) = -\frac{3x^2}{3h^2(x,y) - \frac{1}{\sqrt{h(x,y)}}} \qquad \boxed{\frac{\partial h}{\partial x}(1,0) = -\frac{3}{2}}$

$\frac{\partial F}{\partial y}(x,y,z) = 3y^2 \qquad \frac{\partial h}{\partial y}(x,y) = -\frac{3y^2}{3h^2(x,y) - \frac{1}{\sqrt{h(x,y)}}} \qquad \boxed{\frac{\partial h}{\partial y}(1,0) = 0}$

$$\boxed{\operatorname{grad} h(1,0) = \left(-\frac{3}{2}, 0\right)}$$

② $F(x,y) = y + \sin y - x = 0$ okolí $(0,0)$

a) $\boxed{g(0) = 0}$ $\qquad g(x) + \sin g(x) - x = 0$

DERIVACE (1. ZPŮSOB)

$g'(x) + \cos g(x) \cdot g'(x) - 1 = 0$

$g'(0) + g'(0) - 1 = 0$

$\boxed{g'(0) = \frac{1}{2}}$

$g''(x) - \sin g(x) \cdot (g'(x))^2 + \cos g(x) \cdot g''(x)$

$g''(0) + g''(0) = 0$

$\boxed{g''(0) = 0}$

$g'''(x) - \cos g(x) \cdot (g'(x))^3 - \sin g(x) \cdot 2g'(x) \cdot g''(x) -$

$- \sin g(x) \cdot g'(x) \cdot g''(x) + \cos g(x) \cdot g'''(x) = 0$

$g'''(0) - \frac{1}{8} + g'''(0) = 0$ $\qquad \boxed{g'''(0) = \frac{1}{16}}$

DERIVACE (2. ZPŮSOB)

$\frac{\partial F}{\partial x}(x,y) = -1$ $\qquad g'(x) = \frac{1}{1 + \cos g(x)}$ $\qquad \boxed{g'(0) = \frac{1}{2}}$

$\frac{\partial F}{\partial y}(x,y) = 1 + \cos y$ $\qquad g''(x) = \left[\frac{1}{1+\cos g(x)}\right]' = \frac{\sin g(x) \cdot g'(x)}{(1 + \cos g(x))^2}$

$\boxed{g''(0) = 0}$

$$g'''(x) = \frac{[\cos g(x)(g'(x))^2 + \sin g(x) \cdot g''(x)] \cdot (1 + \cos g(x))^2 + \sin g(x)\, g'(x) \cdot 2(1+\cos g(x)) \cdot \sin g(x)\, g}{(1 + \cos g(x))^4}$$

$\boxed{g'''(0)} = \frac{\frac{1}{4} \cdot 4 + 0}{16} = \boxed{\frac{1}{16}}$

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b) NE $\quad$ c) ANO $\quad$ d) $T_3(x) = g(0) + g'(0) \cdot x + \frac{1}{2} g''(0) x^2 + \frac{1}{6} g'''(0) x$

$$\boxed{T_3(x) = \frac{x}{2} + \frac{1}{96} x^3}$$

(3) $F(x,y,z) = e^{x^2+y^2} + z^2 - 2 = 0$ v okolí $(0,0,1)$

DERIVACE (1. ZPŮSOB)

$k(0,0)=1$

$$e^{x^2+y^2} + k^2(x,y) - 2 = 0$$

$\frac{\partial}{\partial x}$: $e^{x^2+y^2}\cdot 2x + 2k(x,y)\cdot \frac{\partial k}{\partial x}(x,y) = 0$

$e^0 \cdot 2\cdot 0 + 2\cdot \frac{\partial k}{\partial x}(0,0) = 0$

$$\boxed{\frac{\partial k}{\partial x}(0,0) = 0}$$

$\frac{\partial}{\partial y}$: $e^{x^2+y^2}\cdot 2y + 2k(x,y)\cdot \frac{\partial k}{\partial y}(x,y) =$

$$\boxed{\frac{\partial k}{\partial y}(0,0) = 0}$$

$\frac{\partial^2}{\partial x^2}$: $e^{x^2+y^2}\cdot 4x^2 + e^{x^2+y^2}\cdot 2 + 2\left(\frac{\partial k}{\partial x}(x,y)\right)^2 + 2k(x,y)\cdot \frac{\partial^2 k}{\partial x^2}(x,y) = 0$

$$\boxed{\frac{\partial^2 k}{\partial x^2}(0,0) = -1}$$

$\frac{\partial^2}{\partial y^2}$: $e^{x^2+y^2}\cdot 4y^2 + e^{x^2+y^2}\cdot 2 + 2\left(\frac{\partial k}{\partial y}(x,y)\right)^2 + 2k(x,y)\ \frac{\partial^2 k}{\partial y^2}(x,y) = 0$

$$\boxed{\frac{\partial^2 k}{\partial y^2}(0,0) = -1}$$

$\frac{\partial^2}{\partial x \partial y}$: $e^{x^2+y^2}\cdot 2x 2y + 2\frac{\partial k}{\partial y}(x,y)\frac{\partial k}{\partial x}(x,y) + 2k(x,y)\cdot \frac{\partial^2 k}{\partial x \partial y}(x,y) = 0$

$$\boxed{\frac{\partial^2 k}{\partial x \partial y}(0,0) = 0}$$

a) $\det\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = 1 > 0 \Rightarrow$ lok. extrém

$\frac{\partial^2 k}{\partial x^2}(0,0) < 0 \Rightarrow$ lok. maximum

b) $\boxed{T_2(x,y) = 1 - \frac{x^2}{2} - \frac{y^2}{2}}$

$k(0{,}1; -0{,}1) \doteq T_2(0{,}1; -0{,}1) = 1 - \frac{0{,}01}{2} - \frac{0{,}01}{2} = \underline{\underline{0{,}99}}$

③ DERIVACE (DRUHÝ ZPŮSOB)

$$\frac{\partial F}{\partial x}(x,y,z) = e^{x^2+y^2}\cdot 2x$$

$$\frac{\partial F}{\partial y}(x,y,z) = e^{x^2+y^2}\cdot 2y$$

$$\frac{\partial F}{\partial z}(x,y,z) = 2z$$

$$\frac{\partial k}{\partial x}(x,y) = -\frac{e^{x^2+y^2}\cdot 2x}{2\cdot k(x,y)}$$

$$\frac{\partial k}{\partial y}(x,y) = -\frac{e^{x^2+y^2}\cdot y}{k(x,y)}$$

$$\boxed{\frac{\partial k}{\partial x}(0,0)=0} \quad \boxed{\frac{\partial k}{\partial y}(0,0)=0}$$

$$\frac{\partial^2 k}{\partial x^2}(x,y) = \frac{\partial}{\partial x}\left[-\frac{e^{x^2+y^2}\cdot x}{k(x,y)}\right] = \frac{\left[-e^{x^2+y^2}\cdot 2x^2 - e^{x^2+y^2}\right]k(x,y) + e^{x^2+y^2}\cdot x\,\frac{\partial k}{\partial x}(x,y)}{k^2(x,y)}$$

$$\boxed{\frac{\partial^2 k}{\partial x^2}(0,0) = -1}$$

$$\frac{\partial^2 k}{\partial y^2}(x,y) = \frac{\left[-e^{x^2+y^2}\cdot 2y^2 - e^{x^2+y^2}\right]k(x,y) + e^{x^2+y^2}\cdot y\cdot\frac{\partial k}{\partial y}(x,y)}{k^2(x,y)}$$

$$\boxed{\frac{\partial^2 k}{\partial y^2}(0,0) = -1}$$

$$\frac{\partial^2 k}{\partial x\partial y}(x,y) = \frac{\left[-e^{x^2+y^2}\cdot 2xy\right]\cdot k(x,y) + e^{x^2+y^2}\cdot x\cdot\frac{\partial k}{\partial y}(x,y)}{k^2(x,y)}$$

$$\frac{\partial^2 k}{\partial y\partial x}(x,y) = \frac{\left[-e^{x^2+y^2}\cdot 2xy\right]k(x,y) + e^{x^2+y^2}\cdot y\,\frac{\partial k}{\partial x}(x,y)}{k^2(x,y)}$$

$$\boxed{\frac{\partial^2 k}{\partial x\partial y}(0,0) = 0} \quad \boxed{\frac{\partial^2 k}{\partial y\partial x}(0,0) = 0}$$