1) $\vec{F}(x,y,z) = \left(\frac{1}{y}, -\frac{x+2z}{y^2}, \frac{2}{y} \right)$

$D(\vec{F}) = \{ (x,y,z) \in \mathbb{R}^3 ; y \neq 0 \}$

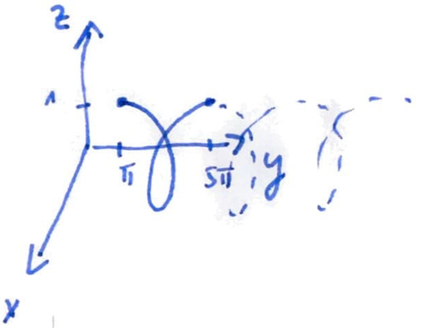
$G = \{ (x,y,z) \in \mathbb{R}^3 ; y > 0 \} \subset D(\vec{F})$

$U(x,y,z) = \frac{x+2z}{y}$

$\operatorname{grad} U(x,y,z) = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right) = \left(\frac{1}{y}, -\frac{x+2z}{y^2}, \frac{2}{y} \right) = \vec{F}(x,y,z)$

$\Rightarrow$ U je potenciálem $\vec{F}$ na G

F je potenciální vekt. pole na G



K: $x = \cos t$

$y = 2t$

$z = \sin t \quad t \in \left\langle \frac{\pi}{2}, \frac{5\pi}{2} \right\rangle$

poč. bod $(0, \pi, 1)$

konc. bod $(0, 5\pi, 1)$

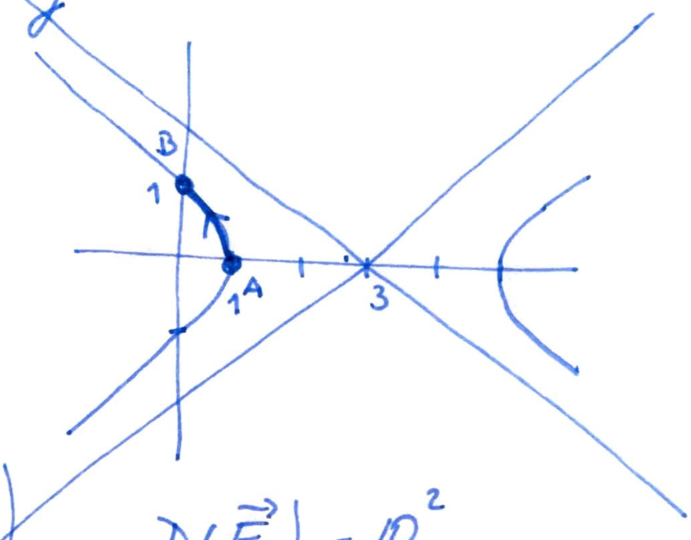
$K \subset G$

$\int_K \vec{F} \cdot d\vec{r}$ nezávisí na cestě

$= U(\text{konc. b.}) - U(\text{poč. b.}) = \frac{2}{5\pi} - \frac{2}{\pi} = -\frac{8}{5\pi}$

(2) $\int_K (x+1)e^{x+y}\,dx + x e^{x+y}\,dy$

$K: \quad (x-3)^2 - 5y^2 = 4$



$A = (1,0)$ poc. b.

$B = (0,1)$ konc. b.

$\vec{F}(x,y) = \left((x+1)e^{x+y}, \; x e^{x+y} \right) \qquad D(\vec{F}) = \mathbb{R}^2$

$\frac{\partial F_1}{\partial y} = (x+1)e^{x+y} \qquad \frac{\partial F_2}{\partial x} = e^{x+y} + x e^{x+y} \qquad \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$ na $\mathbb{R}^2$

$\Rightarrow \vec{F}$ je potenciální na $\mathbb{R}^2$

$K \subset \mathbb{R}^2 \Rightarrow \int_K \vec{F}\cdot d\vec{r}$ nezávisí na cestě

$\boxed{\text{1. ZPŮSOB}}$ – najdu potenciál

$U(x,y) = \int F_2(x,y)\,dy = \int x e^{x+y}\,dy = x e^{x+y} + C(x)$

$\frac{\partial U}{\partial x}(x,y) = e^{x+y} + x e^{x+y} + C'(x) = F_1(x,y)$

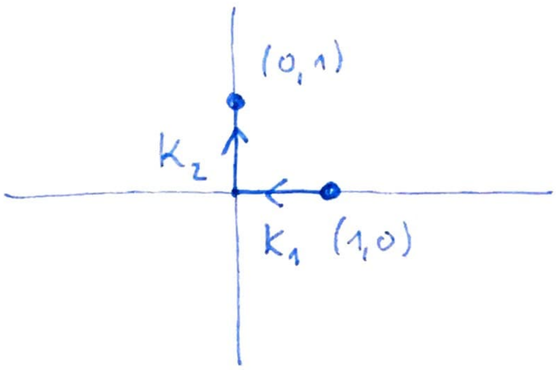
$C'(x) = 0$

$C(x) = C$

$\boxed{U(x,y) = x e^{x+y} + C}$

$\int_K \vec{F}\cdot d\vec{r} = U(0,1) - U(1,0) = \underline{\underline{-e}}$

2. ZPŮSOB – integrál nezávisí na cestě, zvolím pravoúhlou



K_1: $x = -t$ $\quad$ $dx = -1dt$

$y = 0 \quad t \in \langle -1, 0 \rangle \quad dy = 0$

K_2: $x = 0$ $\quad$ $dx = 0$

$y = t \quad t \in \langle 0, 1 \rangle \quad dy = 1dt$

$$\int_K \vec{F} \cdot d\vec{r} = \int_{K_1} \vec{F} d\vec{r} + \int_{K_2} \vec{F} d\vec{r} =$$

$$= \int_{-1}^{0} -(-t+1) e^{-t} dt + 0 + \int_{0}^{1} 0 + 0 \cdot dt =$$

$$= \int_{-1}^{0} (t-1) e^{-t} dt = \left| \begin{matrix} u' = e^{-t} & u = -e^{-t} \\ v = (t-1) & v' = 1 \end{matrix} \right| =$$

$$= \left[-(t-1) e^{-t} \right]_{-1}^{0} + \int_{-1}^{0} e^{-t} dt = \left[-(t-1) e^{-t} - e^{-t} \right]_{-1}^{0}$$

$$= \left[-t e^{-t} \right]_{-1}^{0} = \underline{\underline{-e}}$$

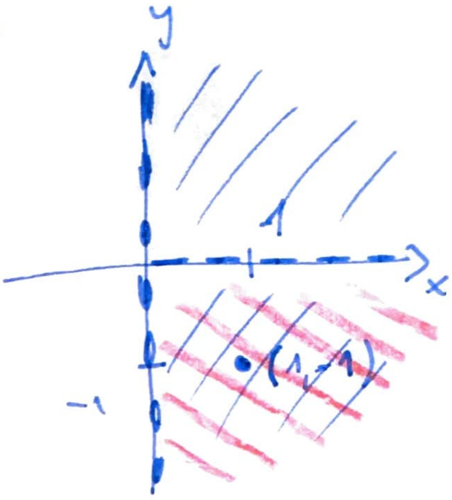
③ $\left(\frac{1}{2\sqrt{x}} - \frac{y}{1+x^2}\right)dx + \left(\operatorname{arccotg} x - \frac{1}{y^2}\right)dy$

$\vec{F}(x,y) = \left(\frac{1}{2\sqrt{x}} - \frac{y}{1+x^2},\ \operatorname{arccotg} x - \frac{1}{y^2}\right)$

$D(\vec{F}) = \{(x,y) \in \mathbb{R}^2,\ x > 0,\ y \neq 0\}$

$\boxed{G = (0,\infty) \times (-\infty, 0)}$ $\quad G \subset D(\vec{F})$

$(1,-1) \in G$ $\quad G$ je konvexná oblasť



$\frac{\partial F_1}{\partial y}(x,y) = -\frac{1}{1+x^2}$

$\frac{\partial F_2}{\partial x}(x,y) = \frac{-1}{1+x^2}$

$\frac{\partial F_1}{\partial y}(x,y) = \frac{\partial F_2}{\partial x}(x,y)$ na G

$\Rightarrow \vec{F}$ je potenciálne na G

$\Rightarrow$ ex. $U(x,y)$ tak, že dif. forma

je ~~to~~ totálnym diferenciálom $U(x,y)$

POTENCIÁL

$U(x,y) = \int F_1(x,y)\,dx = \sqrt{x} + y \operatorname{arccotg} x + C(y)$

$\frac{\partial U}{\partial y}(x,y) = \operatorname{arccotg} x + C'(y) = F_2(x,y)$

$C'(y) = -\frac{1}{y^2} \qquad C(y) = \frac{1}{y} + C$

$U(x,y) = \sqrt{x} + y \operatorname{arccotg} x + \frac{1}{y} + C$

$U(1,-1) = 1 - \frac{\pi}{4} - 1 + C = \frac{\pi}{2} \qquad C = \frac{3\pi}{4}$

$\boxed{U(x,y) = \sqrt{x} + y \operatorname{arccotg} x + \frac{1}{y} + \frac{3\pi}{4}}$