Varianta A

(1.) $f(x) = \dfrac{1}{4^x - 8}$

$$4^x - 8 \neq 0$$
$$4^x \neq 8$$
$$2^{2x} \neq 2^3$$
$$2x \neq 3$$
$$x \neq \frac{3}{2}$$

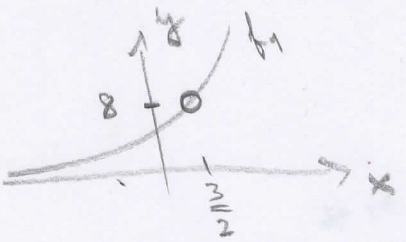
$$\boxed{D_f = \mathbb{R} \setminus \left\{\frac{3}{2}\right\}}$$

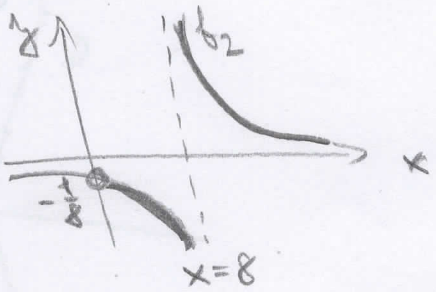
$f(x) = (f_2 \circ f_1)(x)$

$f_1(x) = 4^x$

$f_2(x) = \dfrac{1}{x-8}$

$$\mathbb{R} \setminus \left\{\frac{3}{2}\right\} \xrightarrow{f_1} (0,8) \cup (8,\infty) \xrightarrow{f_2} \left(-\infty, -\frac{1}{8}\right) \cup (0,\infty)$$





$$\boxed{H_f = \left(-\infty, -\frac{1}{8}\right) \cup (0,\infty)}$$

$$y = \frac{1}{4^x - 8}$$

$$y(4^x - 8) = 1$$
$$y4^x - 8y = 1$$
$$4^x = \frac{1+8y}{y}$$
$$x = \log_4\left(\frac{1+8y}{y}\right)$$

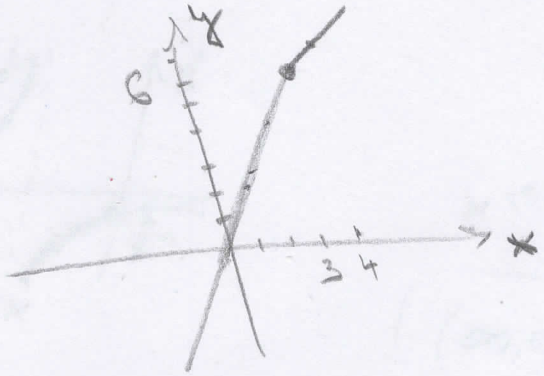
$$\boxed{f^{-1}: y = \log_4\left(\frac{1}{x} + 8\right)}, \ x \in \left(-\infty, -\frac{1}{8}\right) \cup (0,\infty)$$

(2.) $f(x) = \begin{cases} cx, & x \leq 3 \\ \dfrac{x^2-9}{x-3}, & x > 3 \end{cases}$

Máme určit $c \in \mathbb{R}$ tak, aby $\lim\limits_{x \to 3_+} f(x) = f(3)$

$$\lim_{x \to 3_+} f(x) = \lim_{x \to 3_+} \frac{x^2-9}{x-3} = \lim_{x \to 3_+} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 3+3 = 6$$

$$f(3) = c \cdot 3 = 6 \implies c = 2$$



(3.) $f(x) = x^3 - 3x^2 + 4$ $\quad D = \mathbb{R}$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \vee x = 2$

$f' > 0$ | 0 | $f' < 0$ | 2 | $f' > 0$

$f(0) = 4$ ... lok. maximum $[0;4]$

$f(2) = 0$ ... lok. minimum $[2;0]$

$f''(x) = 6x - 6 = 0$

$x = 1$

$\cap$ $\cup$

$f'' < 0$ | 1 | $f'' > 0$

$f(1) = 2$ ... inflexní bod $[1;2]$