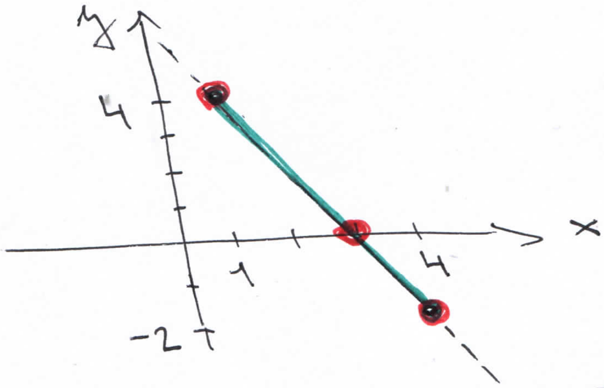
Najděte extrémy funkce f na úsečce s krajními body $C=[1,4]$, $D=[4,-2]$

$$f(x,y)=x^2+y^2+2x+4y$$



$CD \subset g$

$g: \; y = kx + q$

$[1,4]: \; 4 = k\cdot 1 + q \quad | \cdot(-4)$

$[4,-2]: \; -2 = k\cdot 4 + q$

$-18 = -3q \quad | : (-3)$

$\boxed{6 = q}$

$\boxed{k = -2}$

$\boxed{y = -2x + 6}$

$$g(x) := f(x, -2x+6) = x^2 + (-2x+6)^2 + 2x + 4\cdot(-2x+6)$$

$$= x^2 + 4x^2 - 24x + 36 + 2x - 8x + 24$$

$$= 5x^2 - 30x + 60$$

$$g'(x) = 10x - 30$$

$$g'(x) = 0 \iff 10x - 30 = 0$$

$\boxed{x=3}$ $\quad y = -2\cdot 3 + 6$

$\boxed{y = 0}$

body podezřelé z extrému: $[3;0]$, $[4;-2]$, $[1;4]$

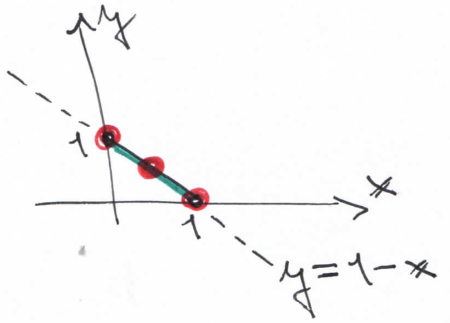
$f(3,0) = 9 + 2\cdot 3 = 15$ - vázané minimum

$f(4,-2) = 16 + 4 + 8 - 8 = 20$

$f(1,4) = 1 + 16 + 2 + 16 = 35$ - vázané maximum

$f(x,y) = x^2 + y^2$

Najděte extrémy funkce f na úsečce AB, $A=[1;0]$
$B=[0;1]$



přímka procházející body A, B:

$y = kx + q$

$A: 0 = k\cdot 1 + q$

$B: 1 = k\cdot 0 + q \implies q=1$
$k=-1$

$g(x) := f(x, 1-x) = x^2 + (1-x)^2 = x^2 + 1 - 2x + x^2$

$= 2x^2 - 2x + 1$

$g'(x) = 4x - 2$

$g'(x) = 0 \iff 4x - 2 = 0 \quad | +2$

$4x = 2 \quad | :4$

$\boxed{x = \frac{1}{2}}$

$y = 1 - x \implies \boxed{y = \frac{1}{2}}$

body podezřelé z extrému: $[\frac{1}{2}; \frac{1}{2}], [1;0], [0;1]$

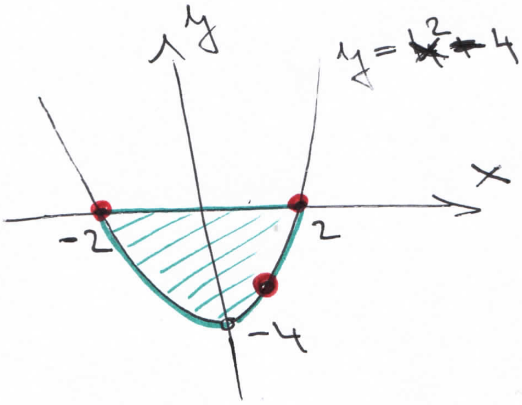
f je spojitá funkce, $M = \{(x,y) \in \mathbb{R}^2 ; y = 1-x,\ x \in [0,1]\}$ omezená uzavřená množina $\implies$ f nabývá na M maxima i minima

$f(\frac{1}{2}, \frac{1}{2}) = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ - vázané minimum

$\left.\begin{array}{l} f(0,1) = 1 \\ f(1,0) = 1 \end{array}\right\}$ vázaná maxima

$f(x,y) = 6x - 3y$

$M = \{(x,y) \in \mathbb{R}^2 \,;\, x^2 - 4 \leq y \leq 0\}$



$\frac{\partial f}{\partial x} = 6 \neq 0$
$\frac{\partial f}{\partial y} = -3 \neq 0$ $\Big\} \Rightarrow f$ nemá stacionární bod.

Hranice: 1) $\underline{y = 0,\ x \in (-2,2)}$: $g(x) := f(x,0) = 6x$

$g'(x) = 6 \neq 0 \quad \forall x \in (-2,2)$

2) $\underline{y = x^2 - 4,\ x \in (-2,2)}$: $g(x) := f(x, x^2-4) =$

$= 6x - 3(x^2-4)$

$= -3x^2 + 6x + 12$

$g'(x) = -6x + 6$

$g'(x) = 0 \iff \boxed{x = 1} \ \boxed{y = -3}$

Body podezřelé z extrému: $[1;-3]$, $[2;0]$, $[-2;0]$

$f(1,-3) = 6 \cdot 1 + 9 = \underline{\underline{15}}$ – vázané maximum

$f(2,0) = 12$

$f(-2,0) = \underline{\underline{-12}}$ – vázané minimum

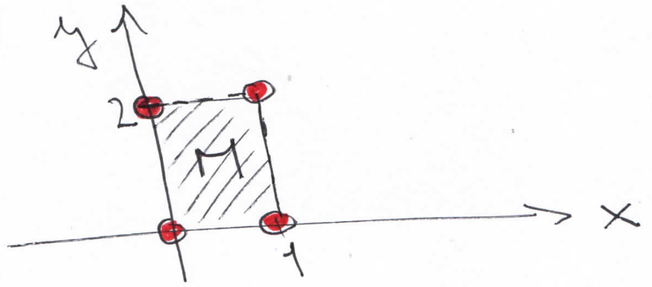
$$f(x,y) = x^2 + 2xy - 4x + 8y$$

$$M = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 1,\ 0 \le y \le 2\}$$

---

f je spojitá funkce, M omezená uzavřená množina

$\Rightarrow f$ nabývá na M maxima i minima



$$\frac{\partial f}{\partial x} = 2x + 2y - 4 = 0$$

$$\frac{\partial f}{\partial y} = 2x + 8 = 0 \iff x = -4,\ y = 6$$

$[-4;6] \notin M$

parametrizace hranice:

1) $y = 0,\ x \in (0,1)$: $g(x) := f(x,0) = x^2 - 4x$

$g'(x) = 2x - 4 = 0 \iff x = 2 \notin (0,1)$

2) $y = 2,\ x \in (0,1)$: $g(x) := f(x,2) = x^2 + 4x - 4x + 16 = x^2 + 16$

$g'(x) = 2x = 0 \iff x = 0 \notin (0,1)$

3) $x = 0,\ y \in (0,2)$: $g(y) := f(0,y) = 8y$

$g'(y) = 8 \neq 0 \quad \forall y \in (0,2)$

4) $x = 1,\ y \in (0,2)$: $g(y) := f(1,y) = -3 + 10y$

$g'(y) = 10 \neq 0 \quad \forall y \in (0,2)$

$f(0,0) = 0$

$f(1,0) = -3$ – vázané minimum

$f(0,2) = 16$

$f(1,2) = 1 + 4 - 4 + 16 = 17$ – vázané maximum

$f(x,y) = x^2 + 4y^2$

$M: \quad x^2 + (y+1)^2 = 4$ – kružnice se středem $[0;-1]$ a poloměrem 2.

$y = -1$

$y = x + 1$

průsečíky křivek:

$x^2 + (x+1+1)^2 = 4$

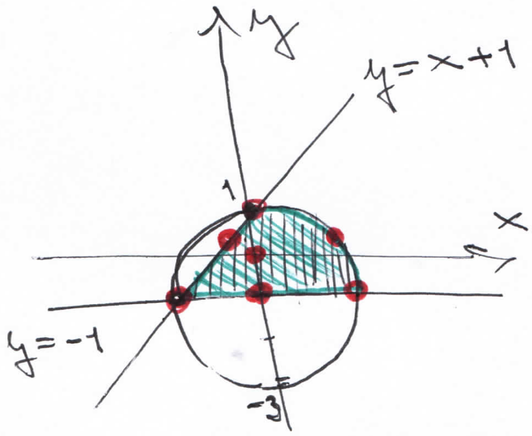
$x^2 + (x+2)^2 = 4$

$x^2 + x^2 + 4x + 4 = 4 \quad | -4$

$2x^2 + 4x = 0$

$2x(x+2) = 0$

$\underline{x=0} \vee \underline{x=-2}$



$\frac{\partial f}{\partial x} = 2x = 0 \iff \underline{x=0}$

$\frac{\partial f}{\partial y} = 8y = 0 \iff \underline{y=0}$

Hranice: 1) $\underline{y = x+1,\ x \in (-2,0)}$

$g(x) := f(x, x+1) = x^2 + 4(x+1)^2 = x^2 + 4(x^2+2x+1)$

$= 5x^2 + 8x + 4$

$g'(x) = 10x + 8 = 0 \iff \boxed{x = -\frac{8}{10}} \boxed{y = \frac{2}{10}}$

2) $\underline{y=-1,\ x \in (-2,2)}$

$g(x) := f(x,-1) = x^2 + 4$

$g'(x) = 2x = 0 \iff \boxed{x=0} \boxed{y=-1}$

3) $\underline{y = \sqrt{4-x^2} - 1,\ x \in (0,2)}$

$g(x) := f(x, \sqrt{4-x^2} - 1) = x^2 + 4 \cdot (4 - x^2 - 2\sqrt{4-x^2} + 1)$

$= -3x^2 - 8\sqrt{4-x^2} + 20$

$g'(x) = -6x + 16x \cdot \frac{1}{2\sqrt{4-x^2}} = 0 \quad |:x \quad | \cdot \sqrt{4-x^2}$

$-6 \cdot \sqrt{4-x^2} + 8 = 0$

$8 = 6 \cdot \sqrt{4-x^2} \quad |:6$

$$\frac{4}{3} = \sqrt{4-x^2} \quad \Big|^2$$

$$\frac{16}{9} = 4 - x^2 \quad \Big| + x^2 \quad \Big| - \frac{16}{9}$$

$$x^2 = \frac{20}{9}$$

$$x = \pm \frac{\sqrt{20}}{3} = \pm \frac{2\sqrt{5}}{3} \qquad y = \frac{1}{3}$$

body podezřelé z extrému: $[-2;-1]$, $[2;-1]$, $[0;1]$, $[0;-1]$, $[0;0]$, $[-\frac{8}{10};\frac{2}{10}]$, $[\frac{\sqrt{20}}{3},\frac{1}{3}]$

$f(-2,-1) = 4+4 = 8$ } vázaná maxima
$f(2,-1) = 8$

$f(0,1) = 4$

$f(0,-1) = 4$

$f(0,0) = 0$ – vázané minimum

$f(-\frac{8}{10},\frac{2}{10}) = \frac{16}{25} + \frac{4}{25} = \frac{20}{25} = \frac{4}{5}$

$f(\frac{\sqrt{20}}{3},\frac{1}{3}) = \frac{20}{9} + \frac{4}{9} = \frac{24}{9}$