$f(x,y) = 6xy - \frac{y}{x} + e^y$

$\frac{\partial f}{\partial x} = 6y + \frac{y}{x^2} = 0 \iff y \cdot \underbrace{\left(6 + \frac{1}{x^2}\right)}_{\neq 0} = 0 \iff y = 0$

$\frac{\partial f}{\partial y} = 6x - \frac{1}{x} + e^y = 0$

dosadíme

$6x - \frac{1}{x} + 1 = 0 \quad / \cdot x$

$6x^2 - 1 + x = 0$

$6x^2 + x - 1 = 0$

$D = 1 - 4 \cdot 6 \cdot (-1) = 25$

$x_{1,2} = \frac{-1 \pm 5}{12} = \begin{cases} -\frac{1}{2} \\ \frac{1}{3} \end{cases}$

Stacionární body: $[-\frac{1}{2}; 0]$, $[\frac{1}{3}; 0]$

$\frac{\partial^2 f}{\partial x^2} = -\frac{y}{x^3}$

$\frac{\partial^2 f}{\partial y^2} = e^y$

$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 6 + \frac{1}{x^2}$

Hessova matice

$H(x,y) = \begin{pmatrix} -\frac{y}{x^3} & 6 + \frac{1}{x^2} \\ 6 + \frac{1}{x^2} & e^y \end{pmatrix}$

$H(-\frac{1}{2}, 0) = \begin{pmatrix} 0 & 10 \\ 10 & 1 \end{pmatrix}$ $\quad \det \begin{pmatrix} 0 & 10 \\ 10 & 1 \end{pmatrix} = -100 < 0$

$\Rightarrow [-\frac{1}{2}, 0]$ je sedlový bod

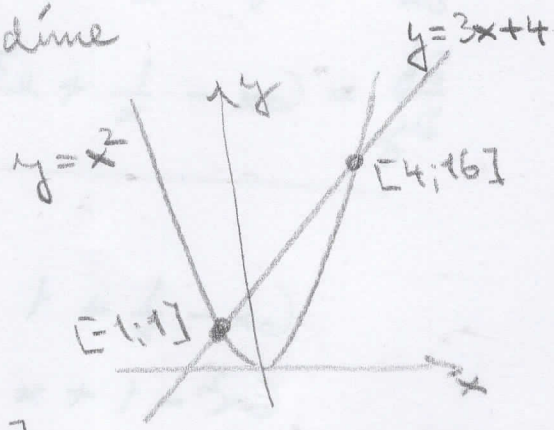
$H(\frac{1}{3}, 0) = \begin{pmatrix} 0 & 15 \\ 15 & 1 \end{pmatrix}$ $\quad \det \begin{pmatrix} 0 & 15 \\ 15 & 1 \end{pmatrix} = -15^2 < 0$

$\Rightarrow [\frac{1}{3}; 0]$ je sedlový bod

$$f(x,y) = x^3 + \frac{1}{2}y^2 - 3xy - 4y$$

$$\frac{\partial f}{\partial x} = 3x^2 - 3y = 0 \iff y = x^2$$

$$\frac{\partial f}{\partial y} = y - 3x - 4 = 0 \quad \text{dosadíme}$$



$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x = 4 \quad \vee \quad x = -1$$

Stacionární body: $[4;16]$, $[-1;1]$

$$\frac{\partial^2 f}{\partial x^2} = 6x$$

$$\frac{\partial^2 f}{\partial y^2} = 1$$

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} = -3$$

Hessova matice:
$$H(x,y) = \begin{pmatrix} 6x & -3 \\ -3 & 1 \end{pmatrix}$$

$$H(4,16) = \begin{pmatrix} 24 & -3 \\ -3 & 1 \end{pmatrix} \qquad \begin{vmatrix} 24 & -3 \\ -3 & 1 \end{vmatrix} = 24 - 9 = 15 > 0$$

$\wedge \; h_{11} = 24 > 0$

$\Rightarrow$ $[4;16]$ je lok. minimum

$$H(-1,1) = \begin{pmatrix} -6 & -3 \\ -3 & 1 \end{pmatrix} \qquad \begin{vmatrix} -6 & -3 \\ -3 & 1 \end{vmatrix} = -6 - 9 = -15 < 0$$

$\Rightarrow$ $[-1,1]$ je sedlový bod