1. Určete a graficky znázorněte definiční obor funkce

   (a) $z = \sqrt{y \sin x}$

   (b) $z = \arcsin(x + y)$

   (c) $z = \frac{x+y-5}{x-y+8}$

   (d) $z = \sqrt{16 - x^2 - 4y^2}$

   (e) $z = \sqrt{y^2 - 1}$

   (f) $z = \ln x + \ln y - \ln(1 - x - y)$

   (g) $z = \ln(y(x + 2))$

   (h) $z = \frac{1}{\arcsin x \cdot \arccos y}$

   (i) $z = (1 - x^2 - y^2)^{-\frac{1}{2}} \arccos 2x$

   (j) $z = \arccos(x - y^2 + 1)$

   (k) $z = z = \sqrt{\cos(2x - y)}$

   (l) $z = \operatorname{tg}(\arcsin(x + y))$

2. Určete definiční obor a nalezněte řez grafu funkce $z = x^2 + 2x + y^2 - 4y + 5$ rovinou $z = 1$.

3. Určete definiční obor a nalezněte řezy grafu funkce $z = x^3 - \sqrt{y}$ souřadnicovými rovinami.

4. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = x^2 + y^2 - 4$.

5. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = 2x + 3y - 4$.

6. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \sqrt{xy}$.

7. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \frac{2x^2}{y^2}$.

8. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \sqrt{9 - x^2 - 9y^2}$.

9. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \ln y^2 - \ln x$.

10. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \frac{5x}{x^2+y^2+1}$.

11. Určete definiční obor a nalezněte vrstevnicový graf funkce $z = \frac{2y}{x^2} + 1$.

12. Vypočtěte následující limity:

    (a) $\lim\limits_{(x,y)\to(-1,6)} \frac{2x+3y-1}{x^4-y}$

    (b) $\lim\limits_{(x,y)\to(\frac{\pi}{4},\frac{\pi}{12})} \sin(x + y) \cdot \cos(x - y)$

    (c) $\lim\limits_{(x,y)\to(-1,4)} \frac{x^3+1}{y(x+1)}$

13. Ukažte, že následující limity neexistují:

    (a) $\lim\limits_{(x,y)\to(0,0)} \frac{x(y+1)}{2x+3y}$

    (b) $\lim\limits_{(x,y)\to(1,4)} \frac{x^2-1}{y-4}$

    (c) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3-y^2}{xy-y^3}$

    (d) $\lim\limits_{(x,y)\to(-1,-3)} \frac{y^3-x^3+26}{x+y+4}$

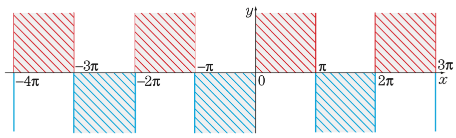
    (e) $\lim\limits_{(x,y)\to(-\frac{1}{2},-\frac{1}{2})} \frac{2x+1}{x+y+1}$

    (f) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+x}{xy+y}$

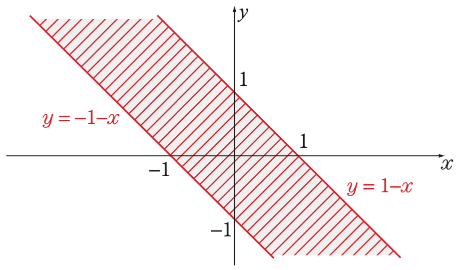
    (g) $\lim\limits_{(x,y)\to(0,0)} \frac{(x^2+y+1)(x^2-y+1)-1}{y}$

Řešení:

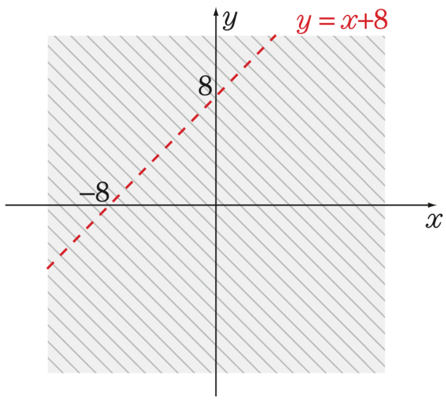
1. (a) $D_z = \Big\{(x,y) \in \mathbb{R}^2 \mid (y \geq 0 \wedge x \in \bigcup_{k\in\mathbb{Z}} \langle 0 + 2k\pi, \pi + 2k\pi\rangle) \vee (y \leq 0 \wedge x \in \bigcup_{k\in\mathbb{Z}} \langle -\pi + 2k\pi,$



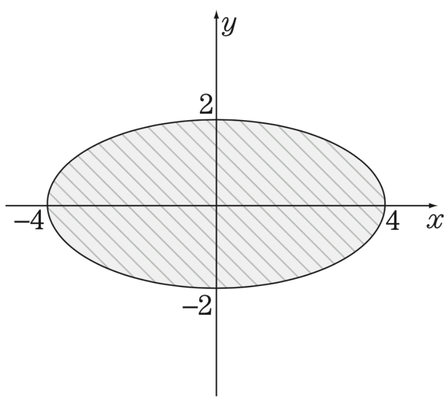
(b) $D_z = \{(x,y) \in \mathbb{R}^2 \mid -1 - x \leq y \wedge y < 1 - x\}$



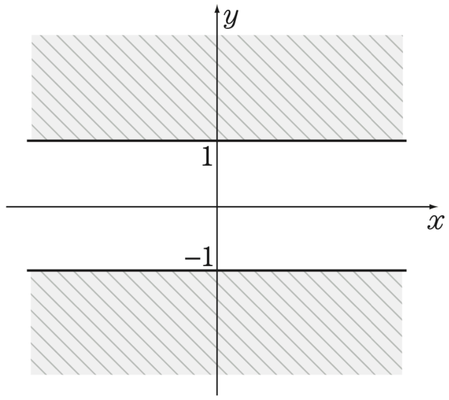
(c) $D_z = \{(x,y) \in \mathbb{R}^2 \mid y \neq x + 8\}$



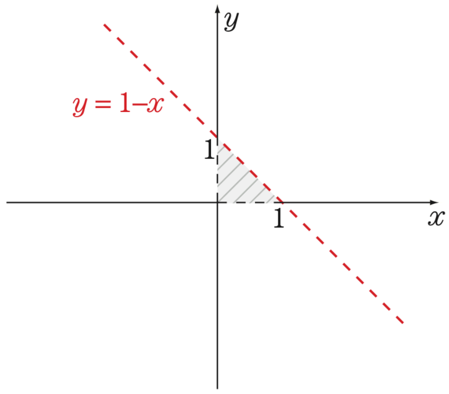
(d) $D_z = \{(x,y) \in \mathbb{R}^2 \mid x^2 + 4y^2 \leq 16\}$



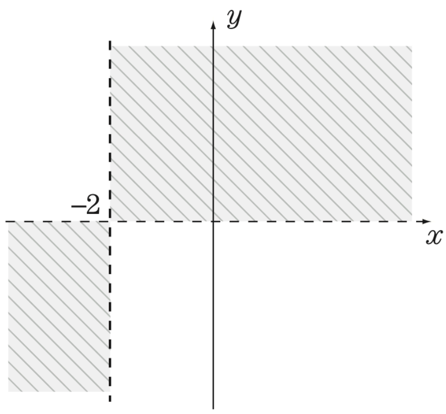
(e) $D_z = \{(x,y) \in \mathbb{R}^2 \mid y^2 \geq 1\}$



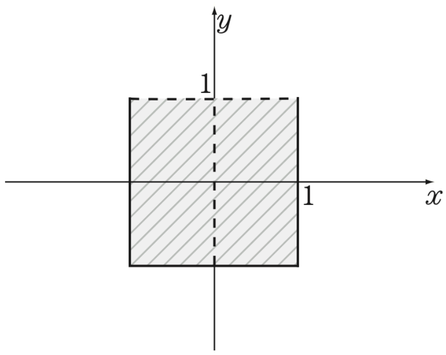
(f) $D_z = \{(x, y) \in \mathbb{R}^2 \mid x > 0 \wedge y > 0 \wedge y < 1 - x\}$



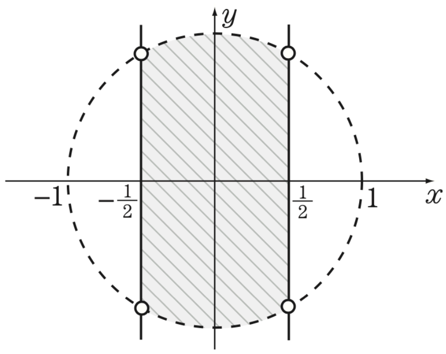
(g) $D_z = \{(x, y) \in \mathbb{R}^2 \mid (y > 0 \wedge x + 2 > 0) \vee (y < 0 \wedge x + 2 < 0)\}$



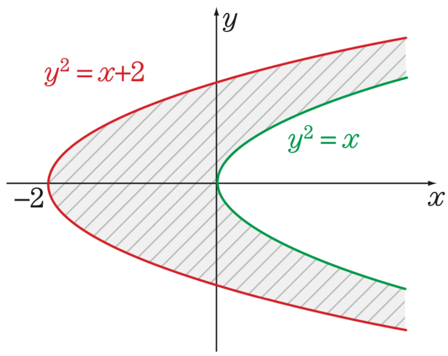
(h) $D_z = \{(x, y) \in \mathbb{R}^2 \mid x \neq 0 \wedge -1 \leq x \leq 1 \wedge -1 \leq y < 1\}$



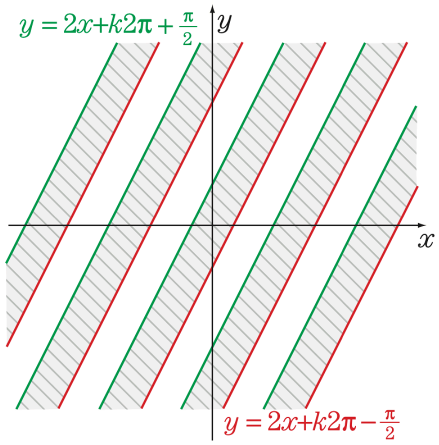
(i) $D_z = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1 \wedge -\frac{1}{2} \leq x \leq \frac{1}{2}\}$



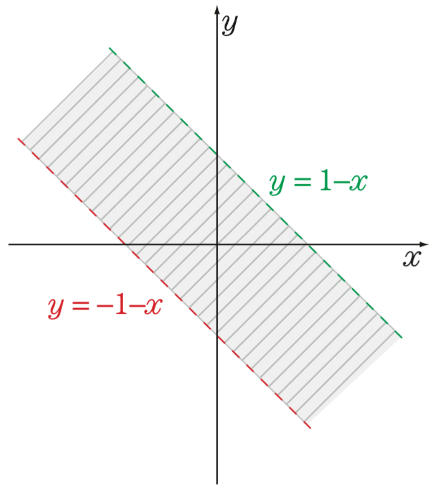
(j) $D_z = \{(x,y) \in \mathbb{R}^2 \mid y^2 \leq x + 2 \wedge x \leq y^2\}$



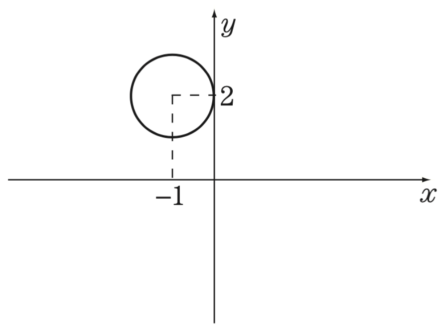
(k) $D_z = \left\{(x,y) \in \mathbb{R}^2 \mid 2x - y \in \bigcup_{k\in\mathbb{Z}} \langle -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi \rangle \right\}$



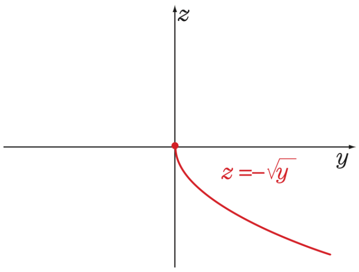
(l) $D_z = \{(x,y) \in \mathbb{R}^2 \mid -1 < x+y < 1\}$

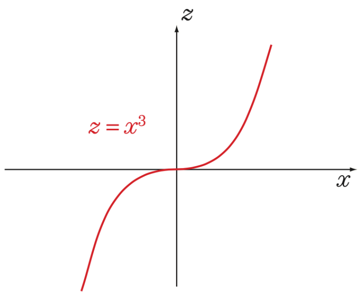


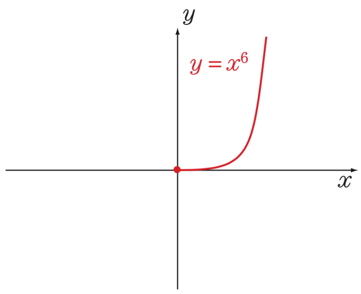
2. $D_z = \mathbb{R}^2$.



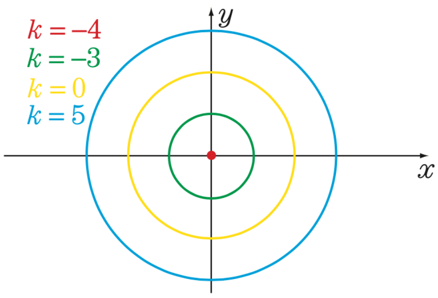
3. $D_z = \{(x,y) \in \mathbb{R}^2 \mid y \geq 0\}$



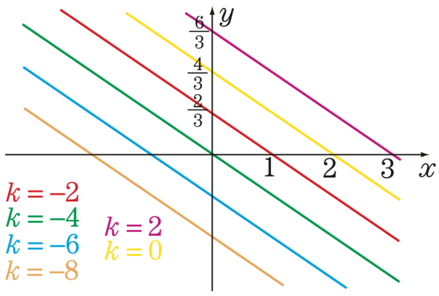




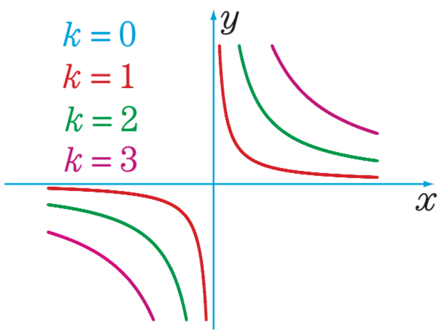
4. $D_z = \mathbf{R}^2$



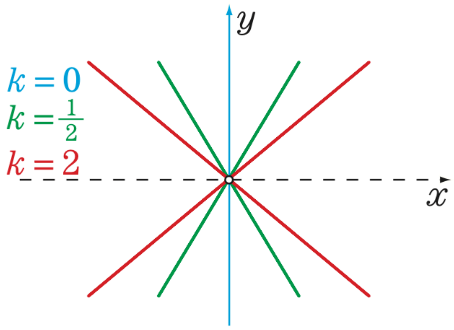
5. $D_z = \mathbf{R}^2$



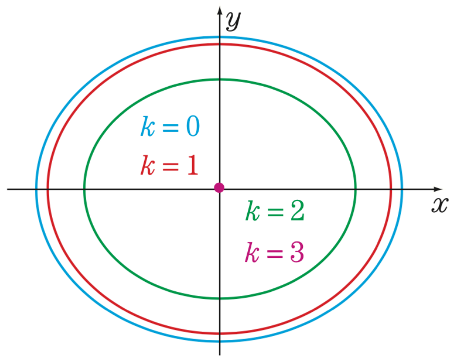
6. $D_z = \{(x, y) \in \mathbb{R}^2 \mid xy \geq 0\}$



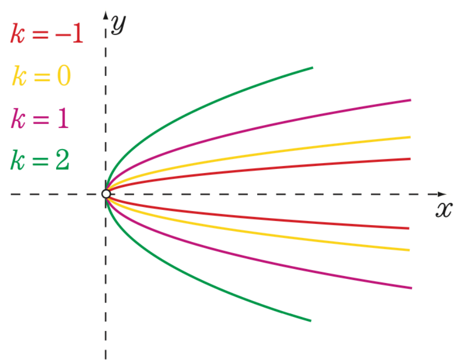
7. $D_z = \{(x, y) \in \mathbb{R}^2 \mid y \neq 0\}$



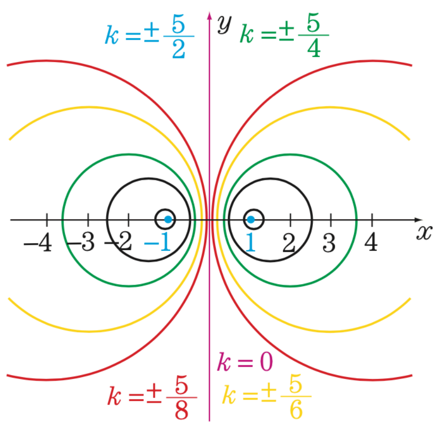
8. $D_z = \{(x, y) \in \mathbb{R}^2 \mid x^2 + 9y^2 \leq 9\}$



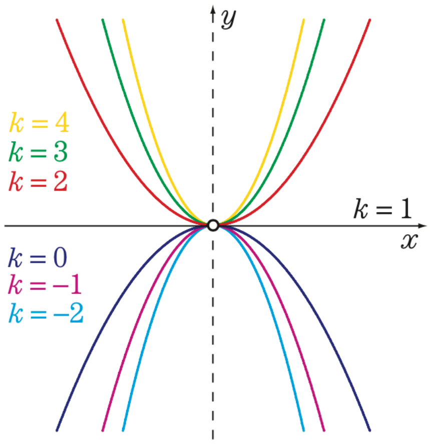
9. $D_z = \{(x, y) \in \mathbb{R}^2 \mid x > 0, y \neq 0\}$



10. $D_z = \mathbb{R}^2$



11. $D_z = \{(x, y) \in \mathbb{R}^2 \mid x \neq 0\}$



12. (a) -3 (b) $\frac{3}{4}$ (c) $\frac{3}{4}$

13. Ukažte, že následující limity neexistují:
    (a) dosazením $y = kx$
    (b) dosazením $y - 4 = k(x - 1)$
    (c) dosazením $y = kx$
    (d) pomocí Věty 1 z přednášky 06
    (e) dosazením $y + \frac{1}{2} = k(x + \frac{1}{2})$
    (f) dosazením $y = kx$
    (g) dosazením $y = kx^2$