5. minitest - varianta A

Vázané extrémy - optimalizace s vazbou

21. 3. 2024

Najděte globální extrémy funkce

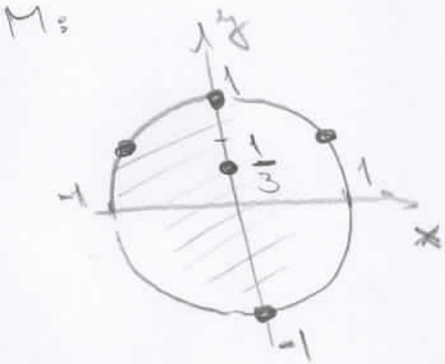
$$f(x,y) = x^2 + 3y^2 - 2y$$

na množině

$$M = \{(x,y) \in \mathbb{R}^2;\ x^2 + y^2 \le 1\}$$

Vnitřek množiny M:

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 2x = 0 \\ \frac{\partial f}{\partial y} &= 6y - 2 = 0 \end{aligned}\right\} \left[0, \tfrac{1}{3}\right] \in M$$



Hranice množiny M: $g(x,y) := x^2 + y^2 - 1 = 0$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 6y-2 \\ 2x & 2y \end{vmatrix} =$$

$$= 4xy - 2x(6y-2) = -8xy + 4x = 4x(-2y+1) = 0$$

$x = 0 \vee y = \frac{1}{2}$

$y = \pm 1 \qquad x = \pm\frac{\sqrt{3}}{2}$

$f(0,1) = 1$

$f(0,-1) = 5$ maximum

$f(0,\frac{1}{3}) = -\frac{1}{3}$ minimum

$f(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{3}{4} + 3 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2} = \frac{1}{2}$

5. minitest - varianta B

Vázané extrémy - optimalizace s vazbou

21. 3. 2024

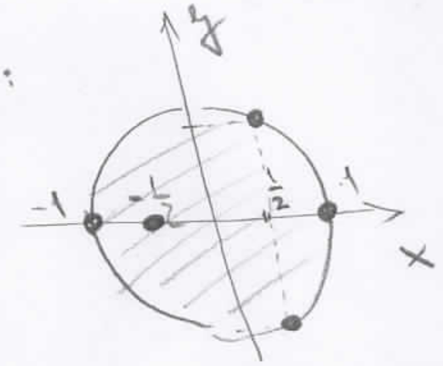
Najděte globální extrémy funkce

$$f(x,y) = x^2 + 2y^2 + x$$

na množině

$$M = \{(x,y) \in \mathbb{R}^2;\ x^2 + y^2 \leq 1\}$$

M:



Vnitřek množiny M:

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 2x+1 = 0 \\ \frac{\partial f}{\partial y} &= 4y = 0 \end{aligned}\right\} \quad [-\tfrac{1}{2}, 0] \in M$$

Hranice množiny M: $\quad g(x,y) := x^2 + y^2 - 1 = 0$

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x+1 & 4y \\ 2x & 2y \end{vmatrix} = 4xy + 2y - 8xy =$$

$$= -4xy + 2y = 2y(-2x+1) = 0$$

$$\Leftrightarrow \quad y = 0 \vee x = \frac{1}{2}$$

$$x = \pm 1 \qquad y = \pm\frac{\sqrt{3}}{2}$$

$f(1,0) = 2$

$f(-1,0) = 0$

$f(-\frac{1}{2},0) = -\frac{1}{4}$ min.

$f(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}) = \frac{1}{4} + 2\cdot\frac{3}{4} + \frac{1}{2} = \frac{9}{4}$ max.